

LOW-COST STOCHASTIC OPTIMIZATION FOR ENGINEERING APPLICATIONS

**A.P. Giotis[†], M. Emmerich[‡], B. Naujoks[‡], K.C. Giannakoglou[†], Th.
Bäck[‡]**

[†]
*Laboratory of Thermal Turbomachines
National Technical University of Athens,
P.O. Box 64069, Athens 15710, Greece
Email: kgianna@central.ntua.gr
agiotis@mail.ntua.gr*

[‡]
*Center for Applied Systems Analysis
Informatik Centrum Dortmund
Joseph v. Fraunhofer Str. 20, 44227
Dortmund, Germany
Email: {emmerich, naujoks}@icd.de
baeck@nutechsolutions.de*

Abstract. This paper presents a technique which when used with Evolutionary Algorithms (Genetic Algorithms, Evolutionary Strategies) reduces noticeably the computational cost by decreasing the number of exact evaluations required to reach the optimal solution. This technique is based on the use of “local” surrogate evaluation models, namely radial basis function networks which are trained and used during the evolution. Two engineering applications, namely the inverse design of an airfoil and the optimization of an optical filter layout, are used to demonstrate the gain offered by the proposed technique.

Key words: Stochastic Optimization, Evolutionary Algorithms, Artificial Neural Networks, Metamodels.

1 EVOLUTIONARY ALGORITHMS – PROS AND CONS

Effective and efficient optimization tools are nowadays needed in all fields of engineering applications. Especially in high-dimensional and multiobjective problems, stochastic optimization methods like Evolutionary Algorithms (EAs)^[1] are nowadays well established. They are robust and may readily accommodate existing analysis tools for the evaluation of candidate solutions. However, compared with traditional methods, EAs are known to need a large number of Objective Function Evaluations (OFEs) prior to reaching the optimal solution. The total computing cost for using EAs depends mainly on that of a single OFE. Thus, the former can be reduced by cutting down the number of OFEs required.

Techniques for reducing the cost of EAs through the use of less exact and thus less computationally demanding OFE models can be classified to those using surrogate physical models (with lower modeling accuracy than the standard OFE tool) and those based on surrogate approximation models (such as Response Surface Methods, RSMs, or Artificial Neural Networks, ANNs). In both of them, a database containing previously or purposely evaluated individuals is to be available. In the

past, ANNs have been used by some of the authors as surrogate models for reducing the cost of Genetic Algorithms (GAs) [2, 3, 4]. The aim of this paper is to extend their use to Evolution Strategies (ESs) [5] as well. The reason is that recent ES variants, (either $(\mu, ?)$ or $(\mu+?)$, where μ and $?$ stand for the number of parental and offspring individuals) with sophisticated step-size control mechanisms to adapt the shape of individual mutation distributions, are robust global optimization procedures and may also benefit from the use of surrogate models.

2 LOW - COST EVOLUTIONARY ALGORITHMS

Both GAs and ESs may incorporate surrogate models in a conceptually similar way. In previously proposed low-cost GAs [2, 3, 4], ANNs were used to single out the most promising population members in each generation, i.e. those that worth undergoing exact evaluations. Such a technique reduces the number of exact OFEs per generation from $?$ to $s \cdot ?$ where $0 < s < 1$ (typically $s \sim 0.1$). Though more generations are required, the gain in computing cost is considerable.

There are good reasons for choosing Radial Basis Function Networks (RBFNs) [6] as surrogate models. In the proposed method, RBFNs are trained “locally” and separately for each new individual using a small number of previously evaluated individuals; these should lie in the vicinity of the new individual according to distances measured in the parametric space. The sets of parametric values paired with the corresponding fitness or cost functions, evaluated through the exact OFE software and used for the training of the RBFNs are restored from a database which is dynamically updated during the evolution. The training and use of a RBFN bears almost zero computing cost since it requires the inversion of a small symmetric matrix and this is carried out once for each new individual even in multi-objective optimization. The algorithm starts with either a randomly generated population that keeps evolving for a few generations using exact OFEs or with some systematically chosen individuals (through the so-called “design of experiments” technique) that are evaluated exactly in order to get database entries.

The use of RBFNs as surrogate models with any population based technique is illustrated in Figure 1.

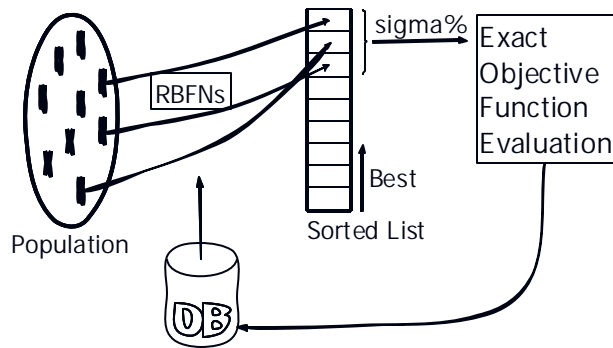


Figure 1: Population-based EAs coupled with RBFNs.

3 APPLICATIONS

3.1 Inverse Design of an Airfoil

In this study, the aim is to find the airfoil shape that produces a given pressure distribution along its contour (Figure 2), at given flow conditions.

The target distribution was calculated in advance through the same software used also for OFEs for a known airfoil profile, thus the goal is indeed to reconstruct the known profile (Figure 3). The parameterization of the airfoil is carried through Bezier curves for the suction and pressure side, with seven control points each, giving a total of 14 design variables.

The gain in the computing cost by employing the surrogate model to GA (with a population size $\mu=40$) or a (5+10)-ES is demonstrated in Figure 4 and Figure 5 respectively for various s values. Each curve is the average of five computations, thus despite the stochastic nature of EAs the reader could place much reliance on that each conclusion drawn is admitting of generalization. Note that in these figures, the horizontal axis stands for the number of OFEs, which is a direct measure of computing cost. In Figure 4, the GA was allowed to run for 2000 OFEs which yielded 50 generations (for $s=100\%$, i.e. the conventional GA) or 240 generations (GA with $s=20\%$) giving an extremely faster convergence in terms of computing cost. The improvement of the ESs was not that pronounced since the convergence of the conventional ES was already satisfactory. In the GA run, many parameters (such as the mutation rate) were not adjusted to optimal values since this requires a lot of experiments. However, the convergence plots for $s=20\%$ were similar.

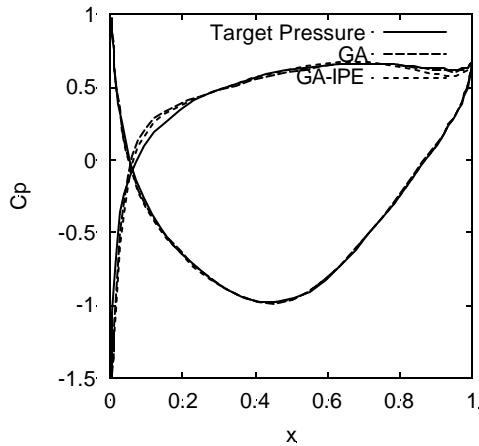


Figure 2: The target and best computed pressure coefficient distributions along the airfoil contour.

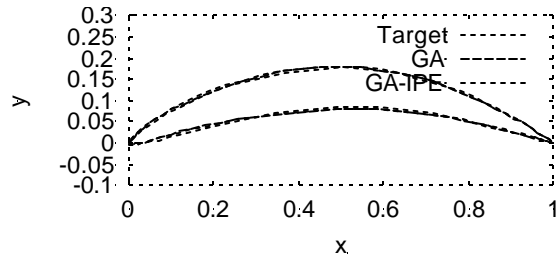


Figure 3: Reconstruction of the target airfoil profile

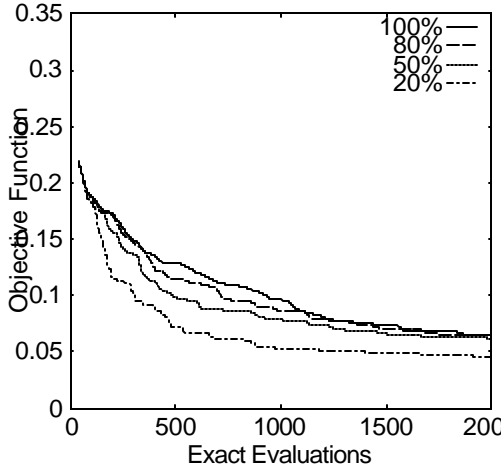


Figure 4: Airfoil Design: Convergence history of the GA for various s values.

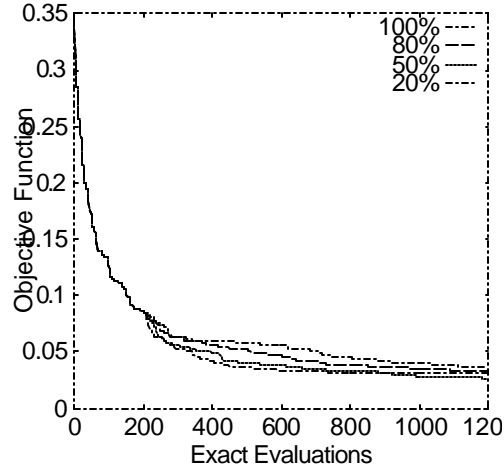


Figure 5: Airfoil Design: Convergence history of a $(5+10)$ -ES for various s values.

3.2 Optical Filter layout optimization

The optical filter test-case proposed by Aguilera et al [7, 8] is a mixed integer optimization problem. The design variables are the thicknesses and the number of up to 21 layers, consisting subsequently of two different substances (germanium and zinc sulfide with refractive indices 4.2 & 2.2 respectively in the wavelength region $7.7-12.3\mu\text{m}$) as shown in Figure 6. The objective is to minimize the reflection mean square function, measuring the distance from a specified target reflectance profile (Figure 7). In this study the mixed integer problem has been re-stated as a continuous problem by assuming that if the thickness of a layer is lower than a small threshold value, this layer is made to disappear completely. This enables a smooth introduction and elimination of layers. The fitness function landscape is multimodal as shown in Figure 8 by limiting the outer left and right optical thicknesses in the range $0-20\mu\text{m}$.

As in the previous test-case, the average convergence histories of 5 computations are illustrated in Figure 9 and Figure 10 for various s values. The population size of the GA was 25 individuals and a $(5+10)$ -ES with local discrete recombination of the object variables and local intermediate recombination of the step-size variables were used. The use of the surrogate model was beneficial for both EAs.

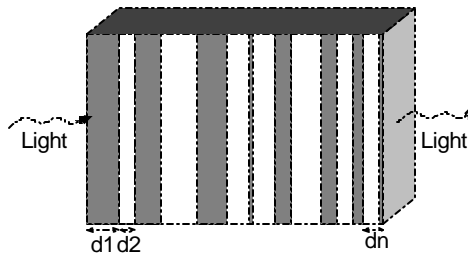


Figure 6: Optical filter with layers of Ge and ZnS

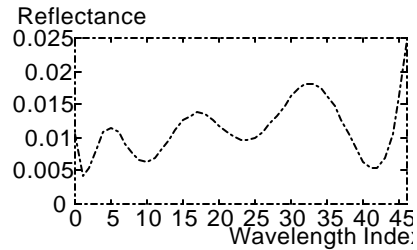


Figure 7: Target reflectance profile

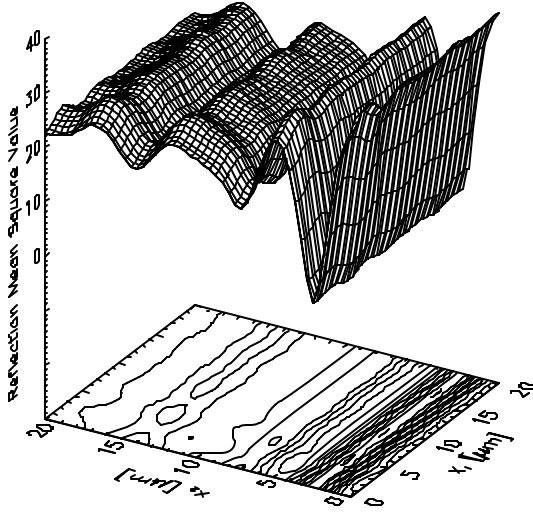


Figure 8: Landscape of the fitness function

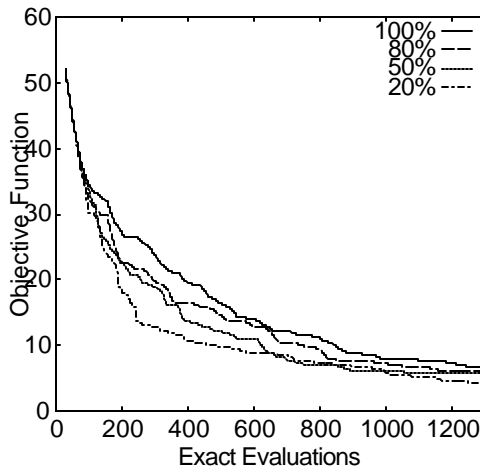


Figure 9: Optical filter: Convergence history of the GA for various s values .

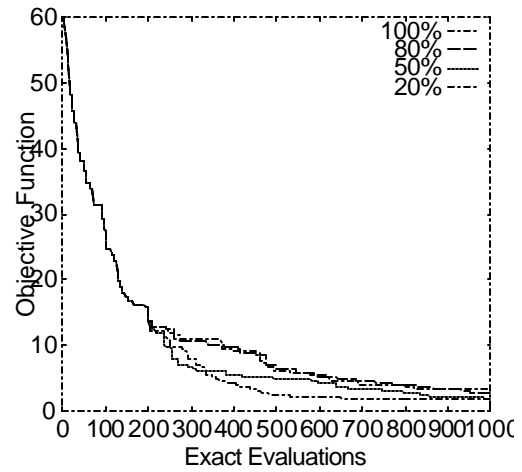


Figure 10: Optical Filter: Convergence history of a (5+10)-ES for various s values.

4 CONCLUSIONS

The scope of this paper was to extend the use of RBFNs as surrogate evaluation models to ES and assess the computational gain in a number of engineering applications. It was concluded that the gain in computing cost is important if only a small part of the population is exactly evaluated whereas all the other are evaluated by the surrogate model. This is appealing since, by doing so, EAs coupled with surrogate models can readily be used in industrial designs, with non-prohibitive computational cost!

Acknowledge ment: The Greek and German groups acknowledge the financial support provided by the IKYDA 2000 project

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