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Continuous Adjoint Methods for Steady and Unsteady Turbulent flows with Emphasis on the Accuracy of Sensitivity Derivatives

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Abstract

The present thesis deals with the mathematical formulations, programming and validation of continuous adjoint methods to steady and unsteady turbulent flows with emphasis on the accuracy of the computed sensitivity derivatives for objective functions related to aerodynamics. Applications include shape and flow-control optimization problems in internal and external flows of both academic and industrial origin.

Regarding the differentiation of turbulence models, the continuous adjoint method is extended to cover flows governed by the $k-\omega$ SST model, for the first time in the relevant literature. The analytical differentiation of the k–ω SST model presents significant difficulties as it includes non-differentiable functions, whose appropriate treatment is presented. The adjoint turbulence model is formulated for both the Low- and the High-Re (using wall functions) variants of the model. Also, the implications of neglecting the differentiation of the turbulence model are investigated. Two distinct adjoint formulations are presented for this turbulence model, namely the surface integrals (*SI*) and the field integrals (*FI*) one. Both formulations are associated with the same adjoint field equations and boundary conditions, though with different expressions for the sensitivity derivatives. In *SI*, only surface integrals are present, whereas, in *FI*, these are expressed in terms of both surface and field integrals.

Between the two adjoint formulations, *SI* and *FI*, numerical discrepancies are observed in the computed sensitivity derivatives which become highly pronounced in non-adequately stretched grids. To identify their origin, the two formulations are compared, both analytically and numerically. As expected, they are mathematically equivalent, but the erroneous handling of a term present in the *SI* formulation, one including the contribution of the grid sensitivities to the sensitivity derivatives is the reason for the computed differences in the sensitivity derivatives. To cope with this problem, a different handling of this term is proposed.

This is capable of computing correct sensitivity derivatives (in agreement with those computed through the *FI* formulation) but its computational cost becomes as high as the latter. This should be avoided, in particular when dealing with industrial applications, and a new adjoint approach is proposed. This is referred to as the Enhanced-*SI* (E-*SI*) adjoint formulation and is able to compute accurate sensitivities with the same computational cost as the *SI* one.

In addition, the adjoint formulation is extended to flows with rotor-stator interaction, using the Multiple-Reference-Frame (MRF) model, commonly referred to as the "frozen rotor" assumption. This adjoint formulation is presented for laminar flows, since its extension to turbulent ones is straightforward for any (already differentiated) turbulence model. Using the efficiency of a turbomachinery stage as the objective function, the proposed adjoint is used to optimize a 2D centrifugal pump with a vaned diffuser.

To accelerate and stabilize the numerical solution of the primal and adjoint equations, the Recursive Projection Method (RPM) is developed in the OpenFoam© environment. Emphasis is laid on the stabilizing capabilities of it when used together with the continuous adjoint solver, especially in flows with small-scale flow unsteadiness, leading to divergence in the numerical solution of the adjoint equations. By identifying and appropriately handling the dominant eigenvalues of the diverging simulation, RPM is able to stabilize the numerical solution. Potential difficulties encountered when using the RPM are discussed.

Using the methods developed above, sensitivity maps are computed over car geometries provided by the automotive industry. In specific, the geometries of the VW L1 concept car and the Audi A7 passenger car are considered. The computed sensitivity maps show to the designer which areas of each car has high optimization potential and how it should be optimized to reduce the exerted drag. A sensitivity map may aid the design of a car even without incorporating it into the optimization loop.

To tackle optimization problems with unsteady flow phenomena, such as vortex generation, the adjoint to the unsteady Navier–Stokes is formulated, for both shape and flow-control optimization problems. The commonly used checkpointing technique is used to overcome the reversed temporal integration encountered in the unsteady adjoint equations. For flow-control applications, the control of the flow around cylindrical geometries using pulsating jets is considered. For shape optimization problems, the centrifugal pump optimized under the “frozen rotor” assumption is re-visited. Using the unsteady flow and adjoint solvers, the flow analysis and the gradient computation are performed without making any relevant assumptions.

Key words: Computational Fluid Dynamics, Continuous Adjoint Methods, Adjoint Turbulence Models, Grid Sensitivities, Reduced Gradient Expressions, Shape and Flow-Control Optimization, Unsteady Adjoint