Robust Design & Adjoint

Requires higher-order derivatives wrt the design and environmental variables which can be efficiently computed using combinations of the Adjoint and Direct Differentiation.

$$\widehat{F} = \hat{\mu_F} + k\hat{\sigma_F}$$

$$\hat{\mu_F} = F_{\mathrm{D}} + \frac{1}{2}\left[\frac{d^2F}{dc_i^2}\right]_{\mathrm{D}}\sigma_i^2$$

$$\hat{\sigma_F} = \sqrt{\left[\frac{dF}{dc_i}\right]_{\mathrm{D}}^2\sigma_i^2 + \frac{1}{2}\left[\frac{d^2F}{dc_idc_j}\right]_{\mathrm{D}}^2\sigma_i^2\sigma_j^2}$$

$$\frac{d\widehat{F}}{db_l} = \frac{dF}{db_l} + \frac{1}{2}\frac{d^3F}{dc_i^2db_l}\sigma_i^2 + k\frac{2\frac{dF}{dc_i}\frac{d^2F}{dc_idb_l}\sigma_i^2 + \frac{d^2F}{dc_idc_j}\frac{d^3F}{dc_idc_jdb_l}\sigma_i^2\sigma_j^2}{2\sqrt{\left[\frac{dF}{dc_i}\right]^2\sigma_i^2 + \frac{1}{2}\left[\frac{d^2F}{dc_idc_j}\right]^2\sigma_i^2\sigma_j^2}}$$

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Computation of Higher-Order Derivatives

$$\frac{dF}{db_i} = \frac{\partial F}{\partial b_i} + \frac{\partial F}{\partial U_k}\frac{dU_k}{db_i}$$

$$\frac{d^2F}{db_i db_j} = \frac{\partial^2 F}{\partial b_i \partial b_j} + \frac{\partial^2 F}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \frac{\partial^2 F}{\partial U_k \partial b_j}\frac{dU_k}{db_i} + \frac{\partial^2 F}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j} + \frac{\partial F}{\partial U_k}\frac{d^2U_k}{db_i db_j}$$

$$\frac{dR_m}{db_i} = \frac{\partial R_m}{\partial b_i} + \frac{\partial R_m}{\partial U_k}\frac{dU_k}{db_i} = 0$$

$$\frac{d^2R_n}{db_i db_j} = \frac{\partial^2 R_n}{\partial b_i \partial b_j} + \frac{\partial^2 R_n}{\partial b_i \partial U_k}\frac{dU_k}{db_j} + \frac{\partial^2 R_n}{\partial U_k \partial b_j}\frac{dU_k}{db_i} + \frac{\partial^2 R_n}{\partial U_k \partial U_m}\frac{dU_k}{db_i}\frac{dU_m}{db_j} + \frac{\partial R_n}{\partial U_k}\frac{d^2U_k}{db_i db_j} = 0$$

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