NATIONAL TECHNICAL UNIVERSITY OF ATHENS (NTUA)
SCHOOL OF MECHANICAL ENGINEERING
PARALLEL CFD & OPTIMI UNIT (PCOpt/NTUA)

Topology Optimisation

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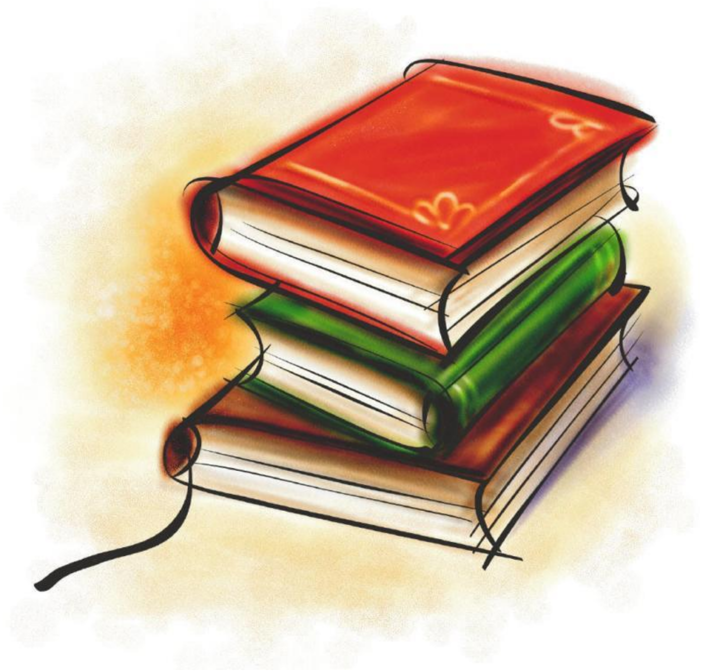
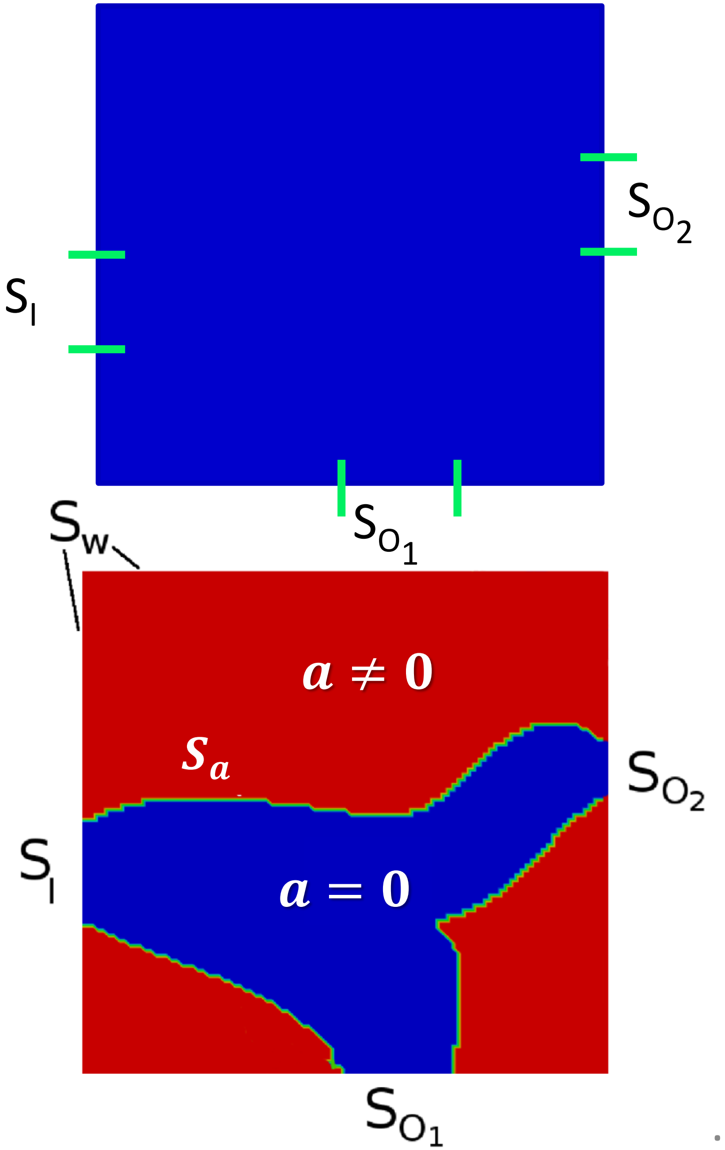


Introduction – denTopO Mathematical Formulation

Topology Optimisation (TopO) in Fluid Mechanics



TopO: seeks optimal porosity or impermeability (α) distribution within a known design space, with fixed inlets and outlets, to minimise the objective function.

Each grid cell is associated with a design (ideally binary) variable:

$\alpha = 1$, cell to be solidified

$\alpha = 0$, cell in which fluid flows

Number of design variables = Number of mesh cells → **Adjoint!**

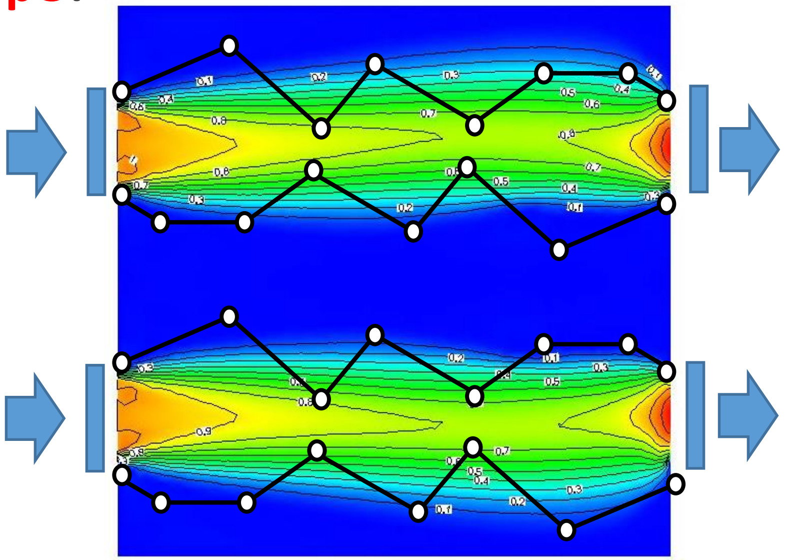
Working with real design variables α, rather than binary, condition $\alpha \neq 0$ should replace $\alpha = 1$!

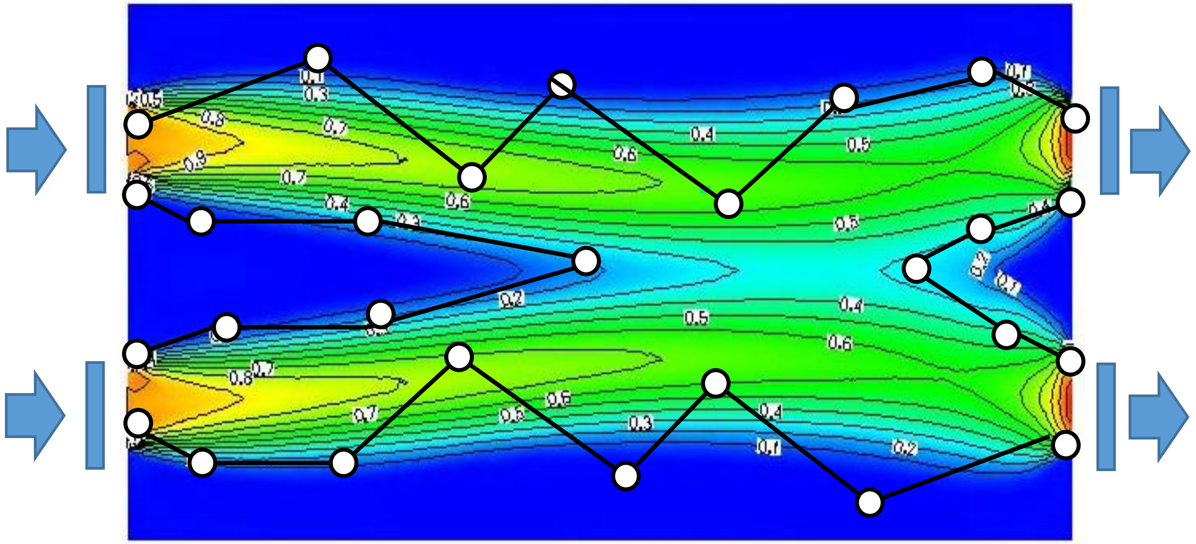
Gray areas between solid and fluid!

Engineering Optimization, 45(8):941-961, 2013.
Archives of Computational Methods in Engineering, 23(2):255-299, 2016.
Computers & Fluids, 150:123-138, 2017.

TopO vs. ShpO in Fluid Mechanics

ShpO starts by parametrising the shape to be optimised and this determines the kind of optimised solutions one may get. Since analysis & optimisation are based on body-fitted grids, accurate boundary conditions are imposed along the FSI (Fluid-Solid Interface) and we, thus, trust the computed performances. Not imposing boundary conditions along the sought FSI is the weak point of (standard) TopO.





Typical examples (short & long design domain) with two inlets (left side) and two outlets (right side).

The Flow Model with Brinkman Penalisation Term (denTopO)

Flow model: RANS equations for incompressible fluid flows, enhanced by *Brinkman* penalization terms modeling solid areas with β~1 as regions almost impermeable to flow. *β* is a field related to the design variables' field *α* .

$$R^p = \frac{\partial v_j}{\partial x_j} = 0$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \beta_{max} I(\beta)\, v_i = 0\ , \quad i = 1,2,3$$

$$R^{\widetilde{\nu}} = v_j \frac{\partial \widetilde{\nu}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{(\nu+\widetilde{\nu})}{\sigma}\frac{\partial \widetilde{\nu}}{\partial x_j}\right) - \frac{c_{b2}}{\sigma}\left(\frac{\partial \widetilde{\nu}}{\partial x_j}\right)^2 + \widetilde{\nu}\left(D(\widetilde{\nu},y) - P(\widetilde{\nu},y)\right) + \beta_{max} I(\beta)\, \widetilde{\nu} = 0$$

$$R^y = \frac{\partial}{\partial x_j}\left(\frac{\partial y}{\partial x_j} y\right) - (1+\epsilon)\, y \frac{\partial^2 y}{\partial x_j^2} - 1 + \beta_{max} I(\beta)\, y = 0$$

- **CHT (Conjugate Heat Transfer) modeling: Solution of the temperature equation *over both the fluid and solid* domains by properly interpolating the thermal conductivity of the two phases.**

$$R^T = \rho^F c_p^F v_j \frac{\partial T}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\left(k(\beta) + (1-\beta)\, k_t\right)\frac{\partial T}{\partial x_j}\right) = 0$$

$$k(\beta) = k_F + (k_S - k_F)\, \beta^p \qquad \textbf{(p=3)}$$

The Flow Model with Brinkman Penalization Terms (denTopO)

The role of the Brinkman Penalisation term(s):

$$R^p = \frac{\partial v_j}{\partial x_j} = 0$$

$$R_i^v = v_j \frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \beta_{max} I(\beta)\, v_i = 0, \quad i = 1,2,3$$

$$R^{\widetilde{\nu}} = v_j \frac{\partial \widetilde{\nu}}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\frac{(\nu + \widetilde{\nu})}{\sigma} \frac{\partial \widetilde{\nu}}{\partial x_j} \right) - \frac{c_{b2}}{\sigma}\left(\frac{\partial \widetilde{\nu}}{\partial x_j} \right)^2 + \widetilde{\nu}\left(D(\widetilde{\nu}, y) - P(\widetilde{\nu}, y)\right) + \beta_{max} I(\beta)\, \widetilde{\nu} = 0$$

$$R^y = \frac{\partial}{\partial x_j}\left(\frac{\partial y}{\partial x_j} y \right) - (1+\epsilon)\, y \frac{\partial^2 y}{\partial x_j^2} - 1 + \beta_{max} I(\beta)\, y = 0$$

The Brinkman penalisation terms impose the wall boundary conditions in a weak sense!

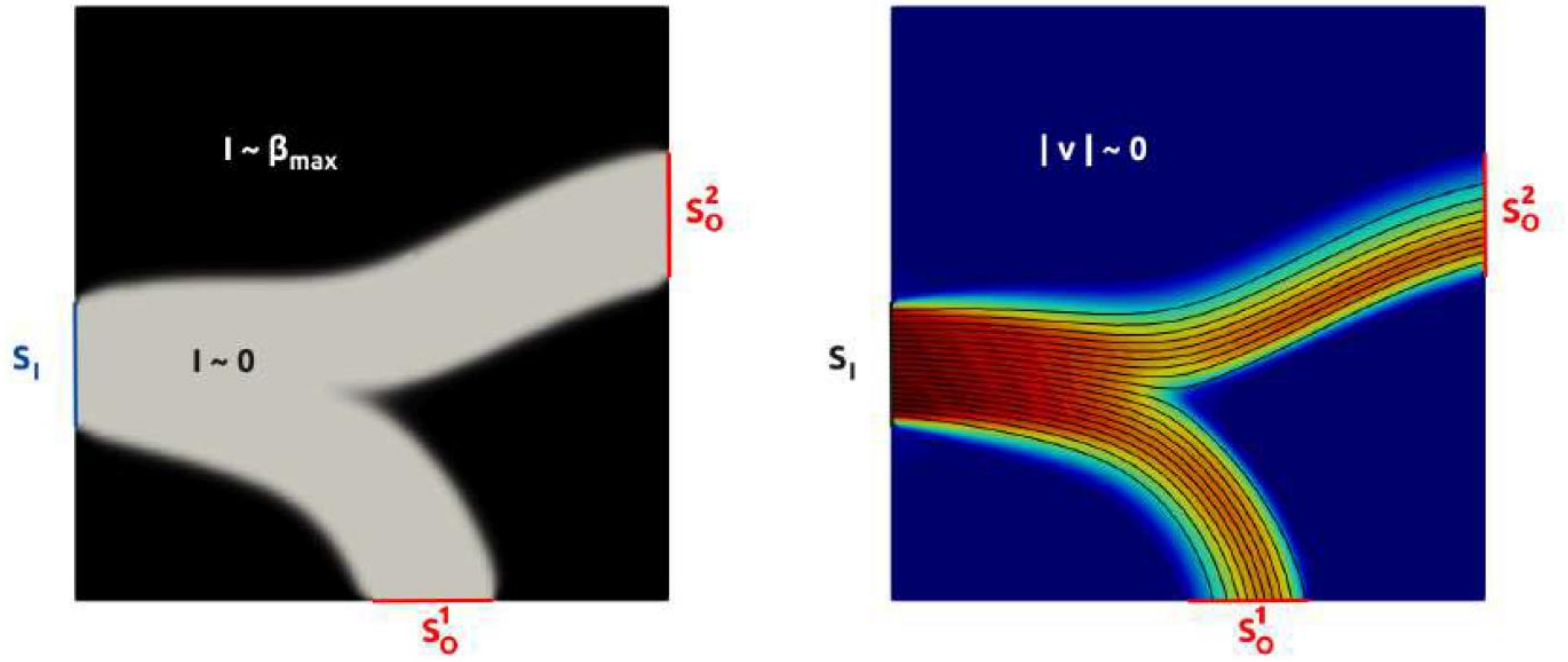
I(β) stands for an impermeability field, depending on the "material" indicator β, given by

$$I_\phi(\beta) = \frac{1}{1 + b(\beta - 1)} \qquad \text{(b=5...10)}$$

where b is a steepening parameter, scaled also by the "max." impermeability of the material (β_{max})

$$\beta_{max} = \frac{\nu}{L^2 Da} \qquad \text{(Da=}10^{-5}\text{)}$$

The Flow Model with Brinkman Penalization Terms (denTopO)



Schematic representation of the role of I(β) in a denTopO problem in fluid mechanics, with a square design domain Ω, designing the optimal duct connecting the inlet (S_I) with the two outlets (S_O).

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

Regularisation/Projection in denTopO

Regularisation **of the porosity field through a Helmholtz-like filter PDE**

$$-r^2 \frac{\partial^2 \widetilde{\alpha}}{\partial x_j^2} + \widetilde{\alpha} = \alpha$$

where $\widetilde{\alpha}$ is the regularised porosity field and *r* a regularisation radius.

Why?

a) To obtain a grid-independent solution.

b) To avoid checkerboard patterns, which are particularly common in CHT TopO.

The $\widetilde{\alpha}$ field is then sharpened (*projected*) to obtain an almost binary β distribution through appropriate functions. β is then used to trace the fluid-solid interface (FSI).

$$\boldsymbol{\alpha} \xrightarrow{\textbf{Regularisation}} \widetilde{\boldsymbol{\alpha}} \xrightarrow{\textbf{Projection}} \boldsymbol{\beta}$$

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From the Design Variables α to the Material Indicator β

The material indicator field, β = β (α) is computed by *sharpening* the α field using a smooth Heaviside projection function:

$$\beta\left(\widetilde{\alpha}\right)=\frac{tanh\left(\eta b\right)+tanh\left(b\left(\widetilde{\alpha}-\eta\right)\right)}{tanh\left(\eta b\right)+tanh\left(b(1-\eta)\right)}$$

where η=0.5, and larger values of b result in “more binary β” fields.
For the development of the adjoint method all these formulas need to be differentiated, so for instance:

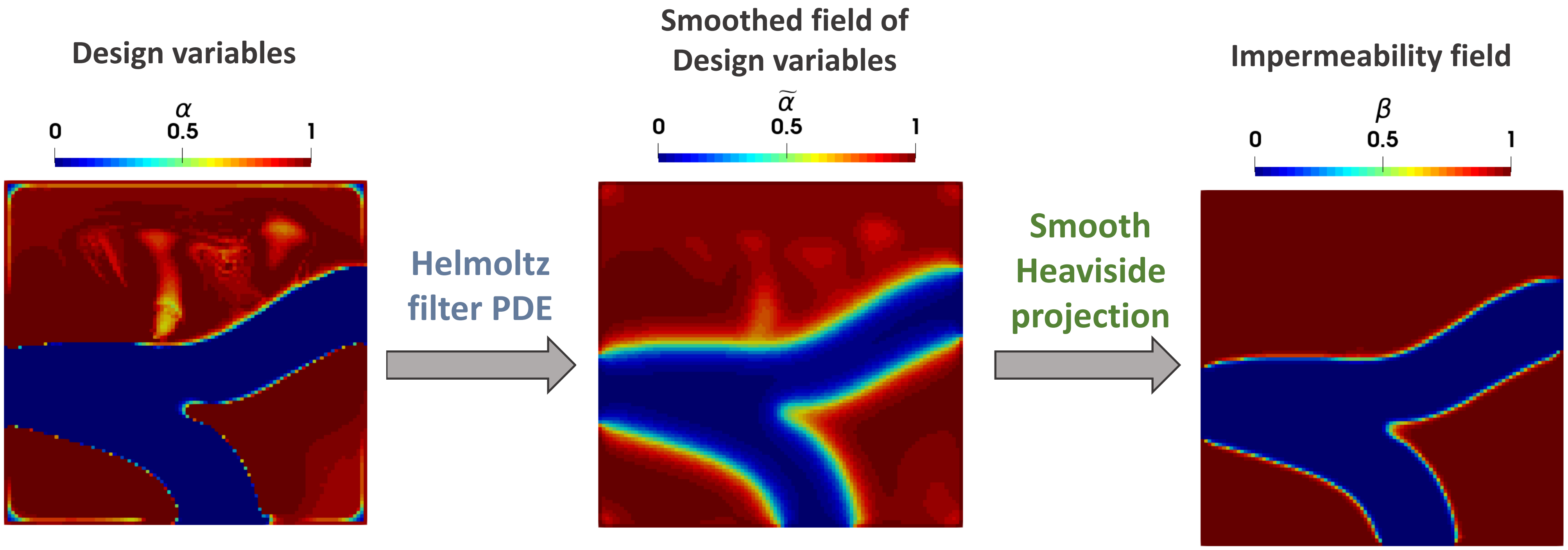
$$\frac{\partial\beta}{\partial\widetilde{\alpha}}=\frac{b\left(1-tanh^2\left(b\left(\widetilde{\alpha}-\eta\right)\right)\right)}{tanh\left(\eta b\right)+tanh\left(b(1-\eta)\right)}$$

For the rest, the chain rule for finding derivatives of a composite function can be used.

F. Wang, B. S. Lazarov, and O. Sigmund. On projection methods, convergence and robust formulations in topology optimization. Struct. Multidisc. Optim., 43:767–784, 2011.

Regularisation/Projection in denTopO

S_{O_1}



- ❑ **β~1 : solid domain**
- ❑ **β~0 : fluid domain**

Schematic representation of the regularisation and projection steps, using typical values: regularisation radius = 10 mesh elements; b=8.

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

Possible Objective (J) & Constraint (g) Functions in denTopO / CHT

$$\mathcal{J}_T = \frac{\int_\Omega \beta T d\Omega}{\int_\Omega \beta d\Omega}$$

Minimise the average T over the solid domain.

$$\mathcal{J}_E = -\int_{S_{I,O}} \rho c_p v_j n_j T dS$$

Minimise the opposite of the enthalpy difference between the fluid's inlet and outlet (=maximise the heat absorbed by the coolant).

$$g_{pt} = \frac{\mathcal{J}_{pt}}{\mathcal{J}_{pt}^{REF}} - 1 \leq 0$$

Keep total pressure losses below a threshold/reference value.

$$g_V = \frac{\int_\Omega (1-\beta)\, d\Omega}{\pi_F \int_\Omega d\Omega} - 1 \leq 0$$

Keep the fluid volume below a certain percentage (π_F) of the total volume.

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The Adjoint Problem

$$\mathcal{L} = J + \int_{\Omega} q R^p d\Omega + \int_{\Omega} u_i R_i^v d\Omega + \int_{\Omega} T_\alpha R^T d\Omega$$

where u_i = adjoint velocity, q = adjoint pressure, and T_α = adjoint temperature

$$R_i^u = -\frac{\partial (v_j u_i)}{\partial x_j} + u_j \frac{\partial v_j}{\partial x_i} + \frac{\partial q}{\partial x_i} - \frac{\partial \tau_{ij}^\alpha}{\partial x_j} + \rho c_p T_\alpha \frac{\partial T}{\partial x_i} + \beta_{max} I(\beta) u_i = 0 \,, \quad i = 1, 2(,3)$$

$$R^q = \frac{\partial u_j}{\partial x_j} = 0$$

$$R^{T_\alpha} = -\frac{\partial (\rho c_p v_j T_\alpha)}{\partial x_j} - \frac{\partial}{\partial x_j} \left[k(I^k(\beta)) \frac{\partial T_\alpha}{\partial x_j} \right] = 0$$

This is performed as in ShpO; the grid does not though change (partial and total derivatives coincide) and this is quite convenient, since:

$$\frac{\partial}{\partial \alpha^P} \left(\frac{\partial \Phi}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left(\frac{\partial \Phi}{\partial \alpha^P} \right)$$

The Adjoint Problem – Sensitivity Derivatives

$$\frac{\delta \mathcal{F}}{\delta \alpha^P} = \int_\Omega \underbrace{\Psi_\beta \frac{\partial \beta}{\partial \widetilde{\alpha}}}_{\Psi_{\widetilde{\alpha}}} \frac{\partial \widetilde{\alpha}}{\partial \alpha^P} d\Omega$$

Computed/avoided (why?) by solving the adjoint to the Helmholtz filtering PDE, i.e. a PDE expressed in terms of the adjoint variable Ψ_α:
SEE NEXT PAGE

where:

$$\Psi_\beta = \left(u_i v_i + \widetilde{\nu}_a \widetilde{\nu} + y_\alpha y\right) \frac{\partial I(\beta)}{\partial \beta} + \frac{\partial T_\alpha}{\partial x_j} \frac{\partial T}{\partial x_j} \left(\frac{\partial k(\beta)}{\partial \beta} - k_t\right) - T_\alpha Q$$

Q	Q_0	$h\left(T_{ref} - T\right)$
Q_α	0	$-hT_\alpha$

Heat-source terms in the primal and adjoint temperature equations.

Galanos N. “Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent “, PhD, NTUA, 2025.

The Adjoint to the Helmholtz Filtering Equation

$$\frac{\delta \mathcal{F}}{\delta \alpha^P} = \int_\Omega \underbrace{\Psi_\beta \frac{\partial \beta}{\partial \widetilde{\alpha}}}_{\Psi_{\widetilde{\alpha}}} \frac{\partial \widetilde{\alpha}}{\partial \alpha^P} d\Omega$$

$$-r^2 \frac{\partial^2 \widetilde{\alpha}}{\partial x_j^2} + \widetilde{\alpha} = \alpha$$

$$\frac{\delta \mathcal{F}}{\delta \alpha^P} = \int_\Omega \Psi_{\widetilde{\alpha}} \frac{\partial \widetilde{\alpha}}{\partial \alpha^P} d\Omega - \int_\Omega \Psi_\alpha \frac{\partial R^F}{\partial \alpha^P} d\Omega$$

Resulting to an extra adjoint equation:

$$R^F\left(\Psi_\alpha, \Psi_{\widetilde{\alpha}}\right) = -r^2 \frac{\partial^2 \Psi_\alpha}{\partial x_j^2} + \Psi_\alpha - \Psi_{\widetilde{\alpha}} = 0$$

and, finally:

$$\frac{\delta \mathcal{F}}{\delta \alpha^P} = \Psi_\alpha^P \Omega^P$$

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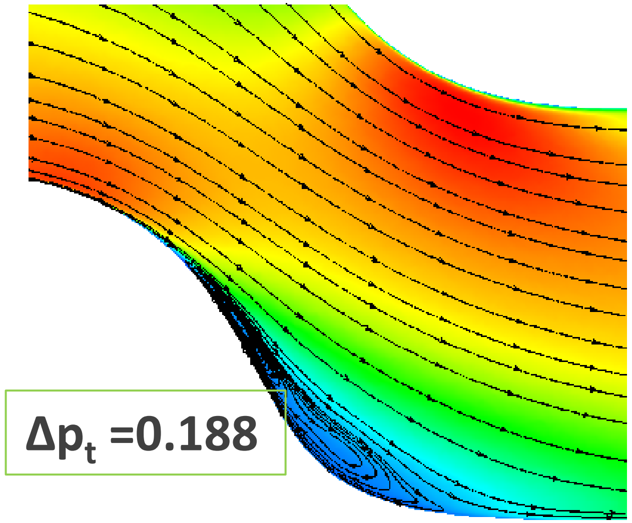
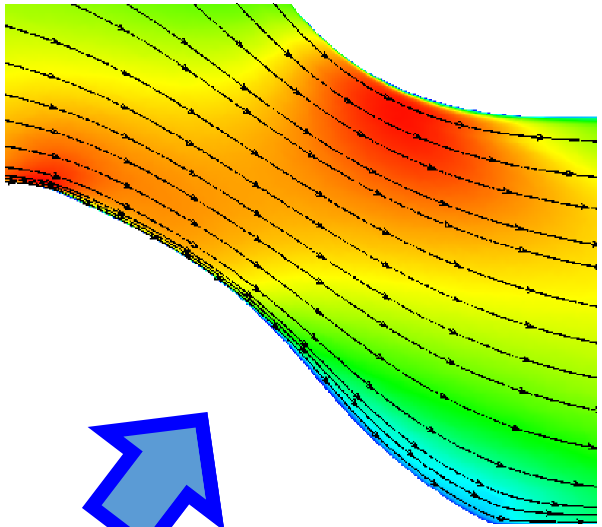
Primal & Adjoint Velocities in TopO – S-Duct, Turbulent Flow

Target: min. Total Pressure Losses (Δp_t)

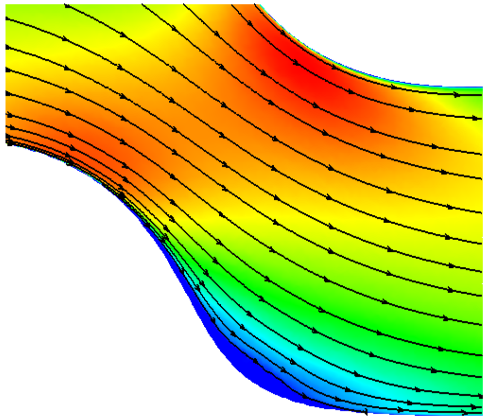
Re = 1.2×10^5



Δp_t =0.188



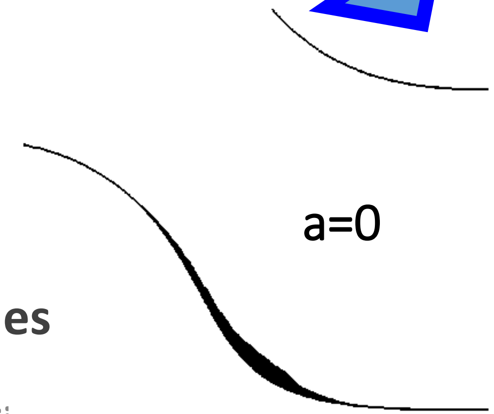
Δp_t =0.158



Δp_t =0.158

15% Reduction in Δp_t

Permeability field

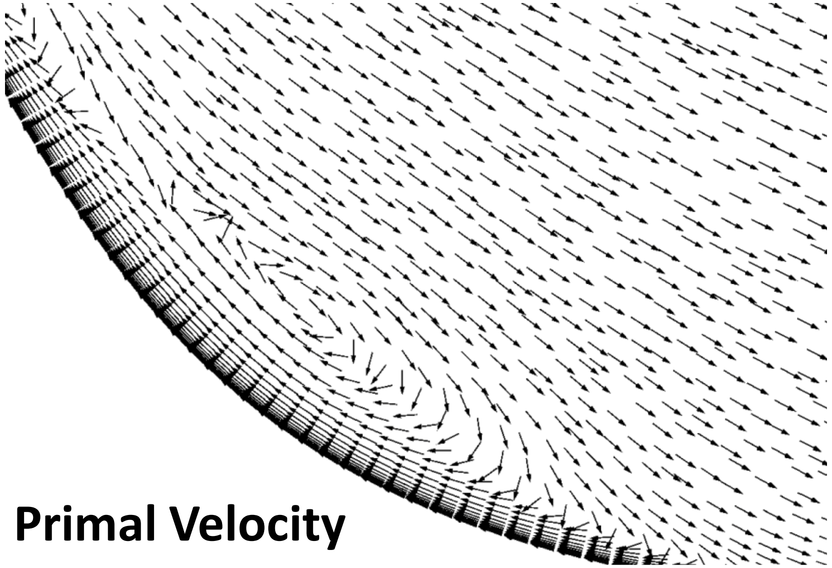


a=0

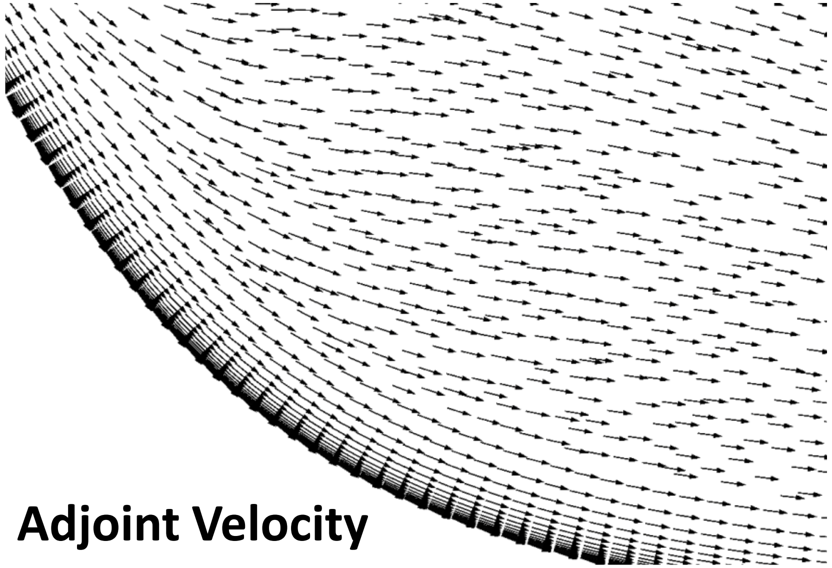
10 TopO Cycles

Prof. K.C. Gi[illegible], [illegible]a.gr, Dr. V.G. Asouti, vasouti@mail.ntua.gr

Primal & Adjoint Velocities in TopO – S-Duct, Turbulent Flow



Primal Velocity



Adjoint Velocity

$$\frac{\delta \mathcal{F}}{\delta \alpha^P} = \int_\Omega \underbrace{\Psi_\beta \frac{\partial \beta}{\partial \widetilde{\alpha}}}_{\Psi_{\widetilde{\alpha}}} \frac{\partial \widetilde{\alpha}}{\partial \alpha^P} d\Omega$$

where:

$$\Psi_\beta = \left(u_i v_i + \widetilde{\nu_a}\widetilde{\nu} + y_\alpha y\right) \frac{\partial I(\beta)}{\partial \beta} + \frac{\partial T_\alpha}{\partial x_j}\frac{\partial T}{\partial x_j}\left(\frac{\partial k(\beta)}{\partial \beta} - k_t\right) - T_\alpha Q$$

TDDC in denTopO

The Think-Discrete-Do-Continuous (TDDC) Adjoint can also be(has been) applied in denTopO (and all other variants of TopO), with the same benefits!

Recall: **The *TDDC* Adjoint bridges the gap between the two adjoint variants and combines the best of both worlds. The concept is to develop consistent discretisation schemes for the continuous adjoint equations, inspired by (hand-differentiated) discrete adjoint.**

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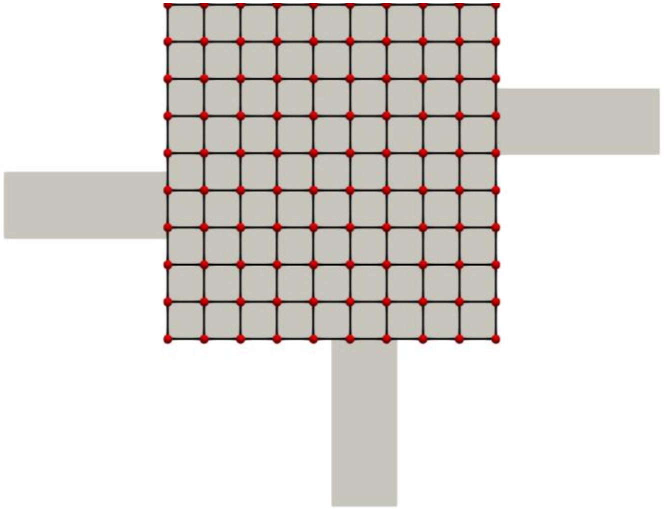
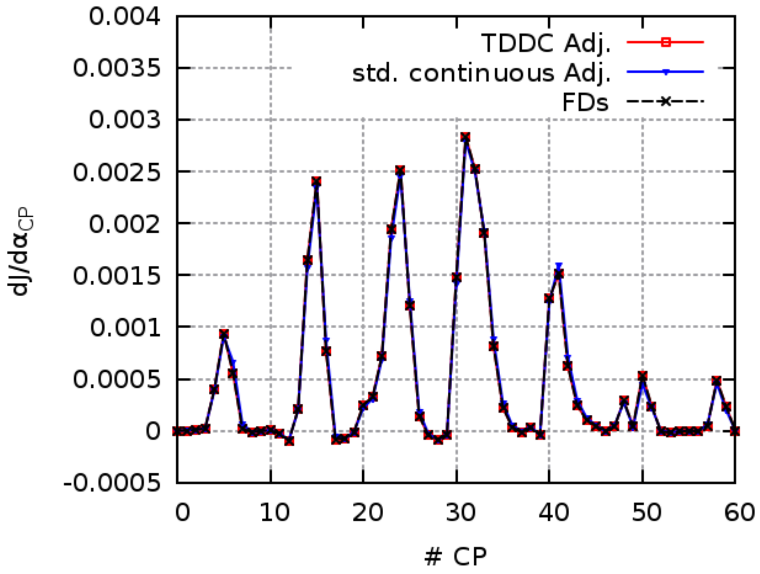
TDDC in denTopO - denTopO of a 1-Inlet-2-Outlet box (1/2)

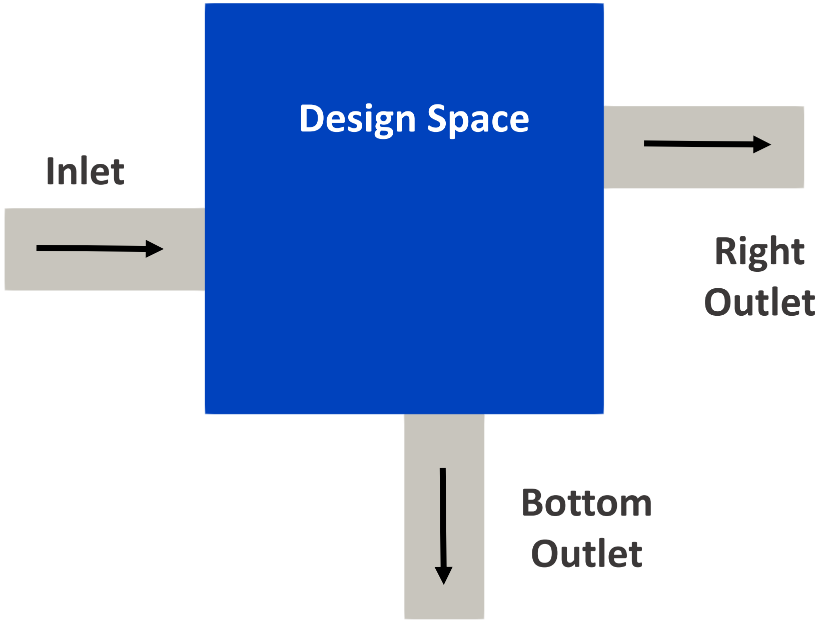
Turbulent case: Re=20000 / CFD Grid with 13K cells
Starting from an all-fluid domain.
Parametrisation/control of the α through a NURBS morphing box.



Parametrisation lattice

SDs of total pressure losses wrt the α values
at the control points (design variables)





Even though both the "standard" and *TDDC* adjoint discretisation schemes provide highly accurate SDs, the latter gives up to 5 significant digit accuracy wrt FDs!

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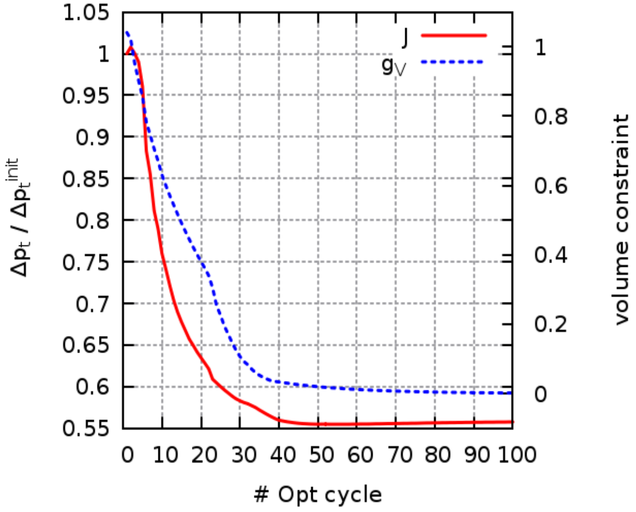
TDDC in denTopO - denTopO of a 1-Inlet-2-Outlet box (2/2)

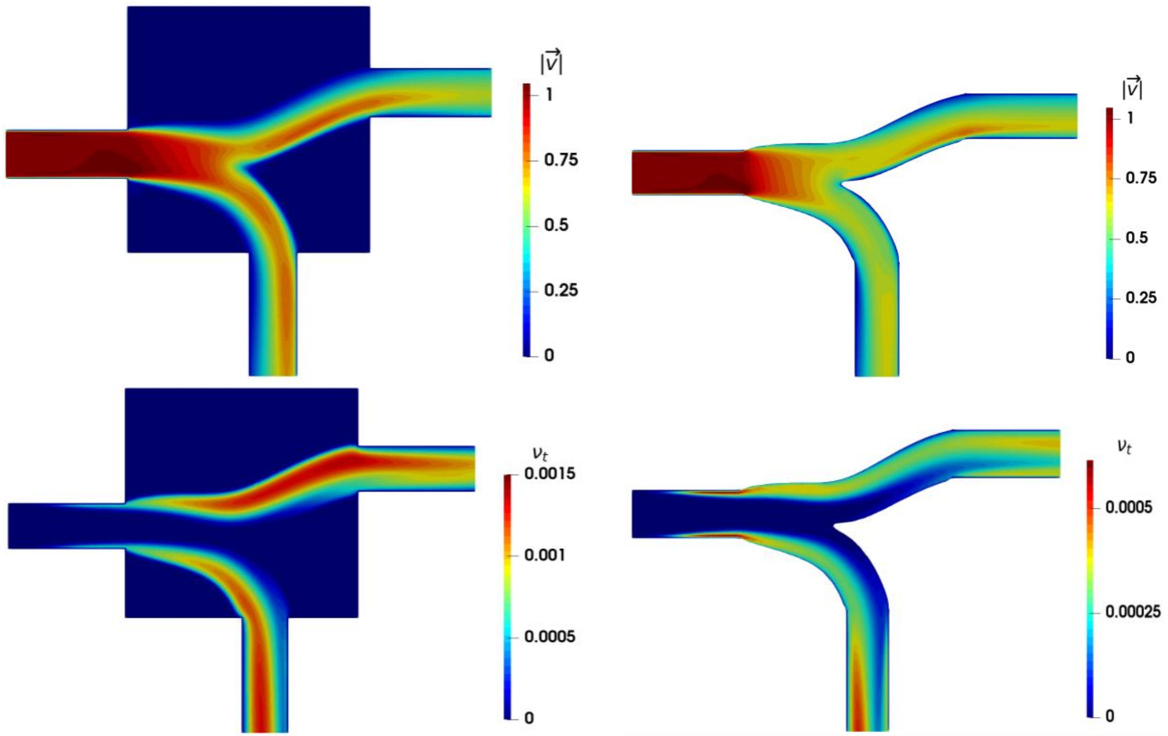
Objective: min. volume-weighted total pressure drop
Constraint: Fluid volume < 46.2% of total volume
Re-evaluation of the *denTopO* solutions on body-fitted grids after extracting the FSI!

$$\Delta p_t = -\int_{S_I^{(\gamma)}} v_j n_j p_t dS - \int_{S_O^{(\gamma)}} v_j n_j p_t dS$$



***denTopO* flow solver** | **Body-fitted grid solver**

Δp_t = 3mW | Δp_t = 0.98mW



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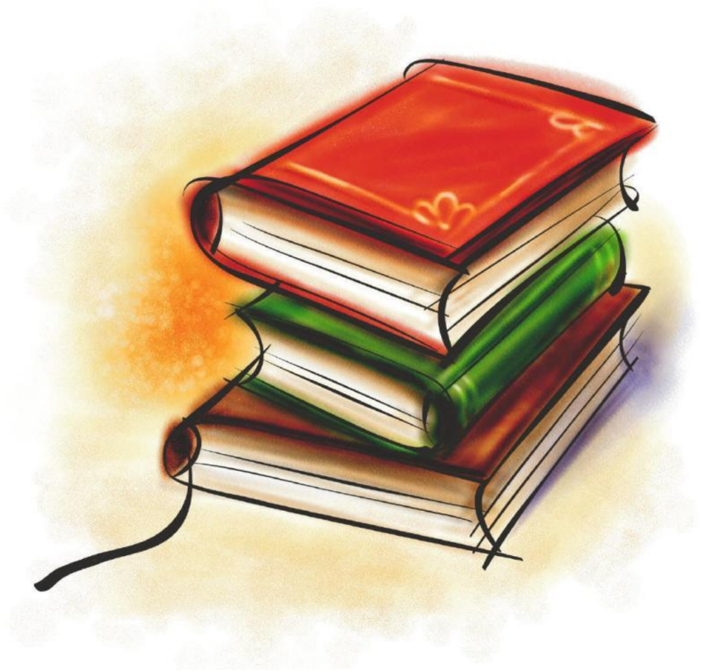


denTopO: Mono-fluid Applications

TopO – Manifold, Turbulent Flow

Targets:

1. Min. Total pressure losses ($f_1 = \Delta p_t$)
2. Max. Heat Transfer ($f_2 = \Delta T_{in/out}$)

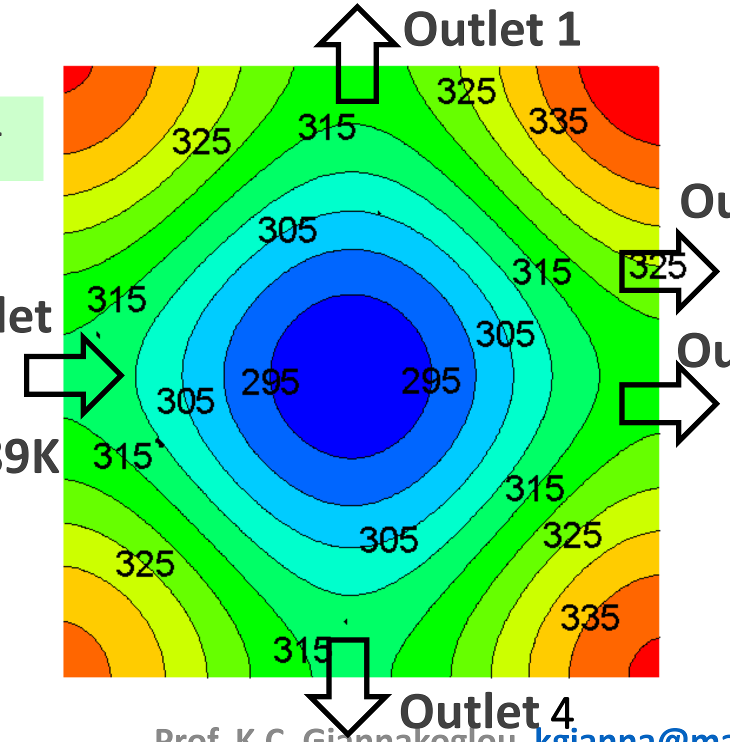
Background Temperature distribution (Twall):

$$T_{wall} = 313 + x^2 - 12cos(\pi x) + y^2 - 12cos(\pi y)$$

Re = $1x10^4$

Inlet

T_{in} =289K



Case 1: Min. Δp_t

Case 2: Min. Δp_t ; constraint:

$m_{out\ 1,2,3,4} = 25\%\ m_{inlet}$

Case 3: Min. Δp_t και Max. ΔT ; constraint :

$m_{out\ 1,2,3,4} = 25\%\ m_{inlet}$

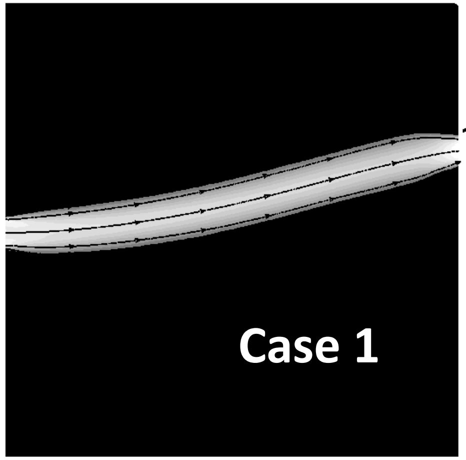
Case 4: Min. Δp_t και Max. ΔT ; constraint :

$T_{out1} = T_{out2} = T_{out3} = T_{out4}$

TopO – Manifold, Turbulent Flow

$f_1=\Delta p_t$
$f_2=\Delta T_{in/out}$

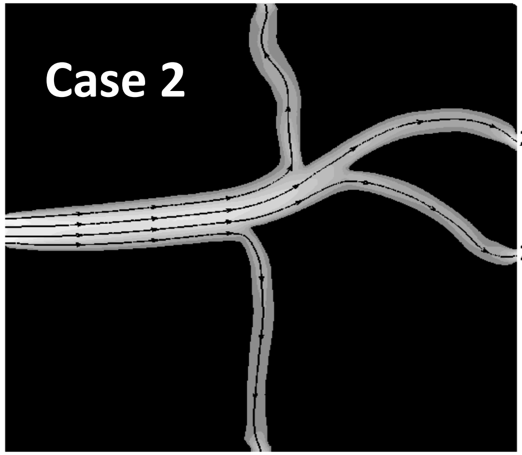
Min. Δp_t



100% - 317K

$f_1=0.025$, $f_2=2.48$

25%-314K

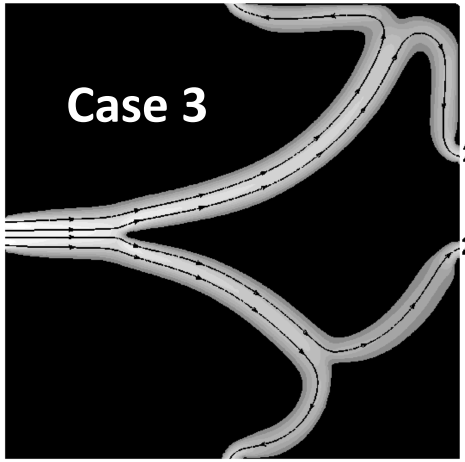


25%-324K
25%-316K
25%-312K

$f_1=0.037$, $f_2=3.00$

Min. Δp_t - Max. ΔT

25%-320K

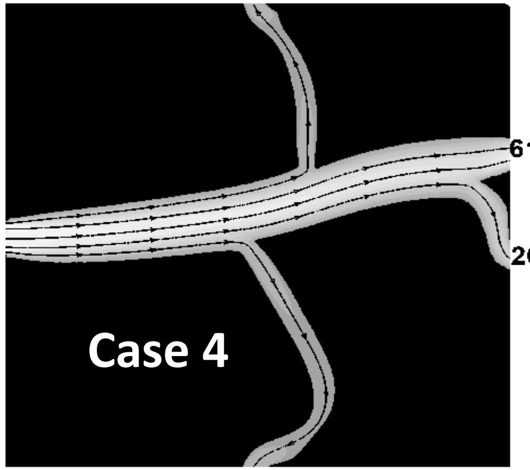


25%-330K
25%-318K
25%-315K

$f_1=0.053$, $f_2=3.51$

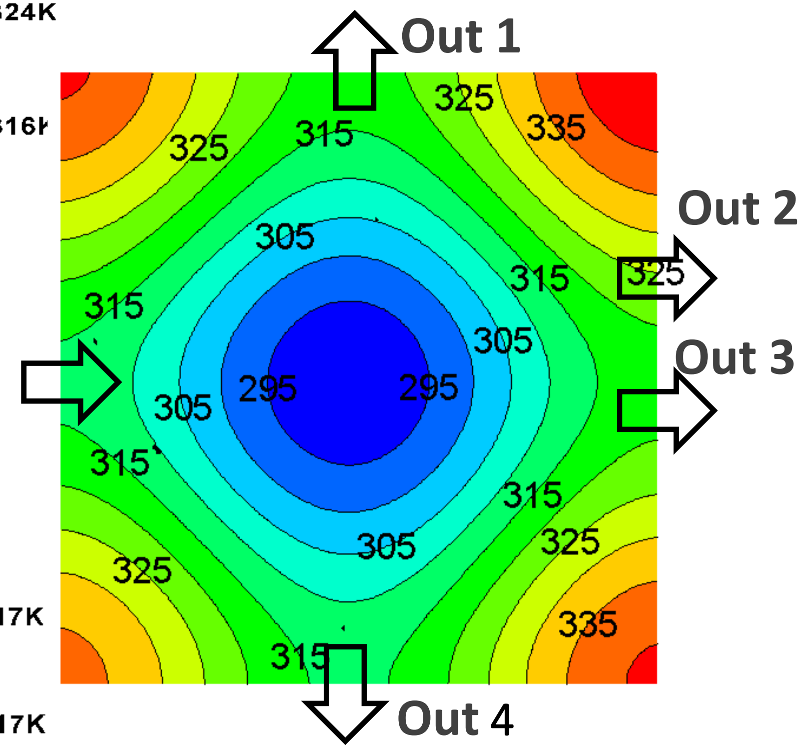
$f_1=0.026$, $f_2=2.83$

9%-317K



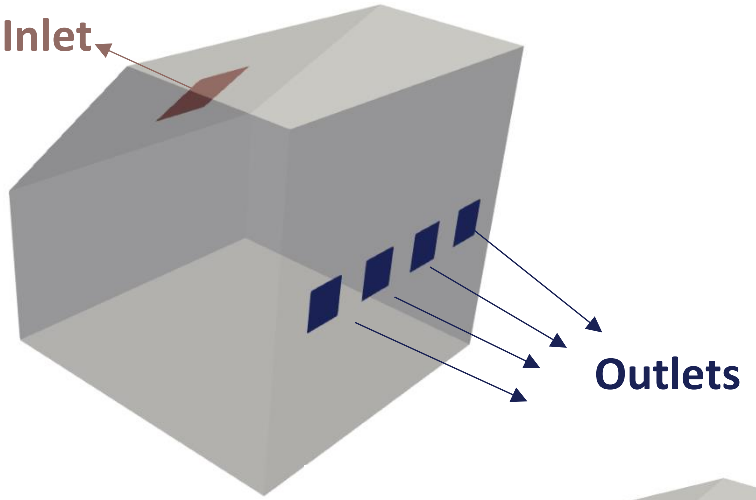
61%-317K
20%-317K
10%-317K

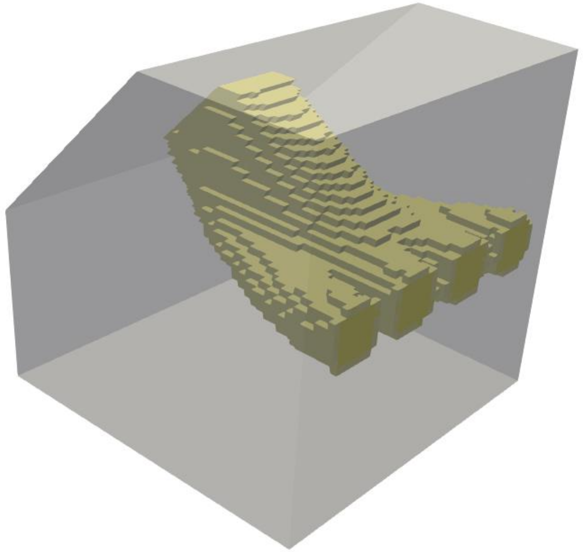
Inlet
T_{in} =289K



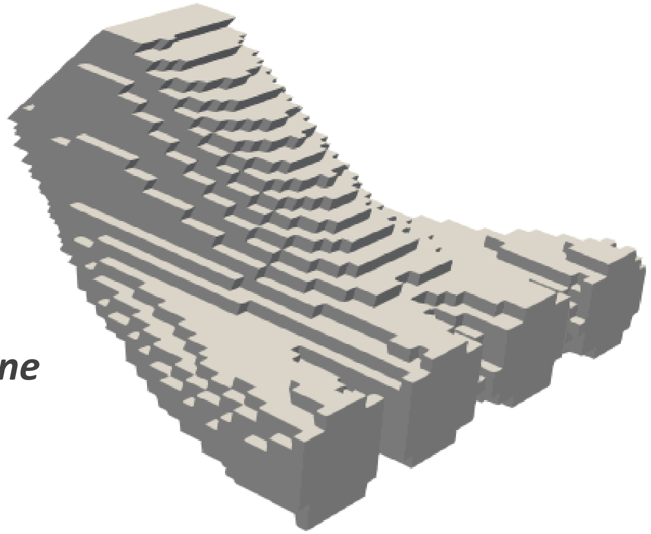
3D Aerodynamic denTopO

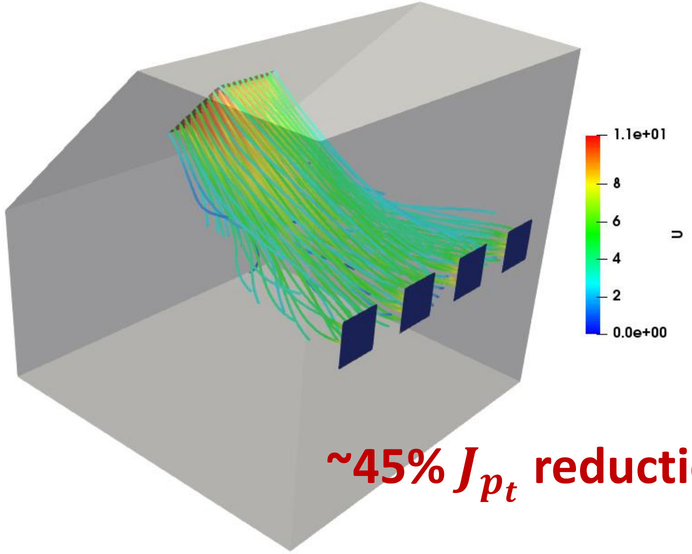
Min. total pressure losses (J_{p_t})
Constraint: Fluid must occupy <30% of the design space
Re = 2000 (laminar flow)
125000 design variables (all mesh cells)





"Zero" β isoline



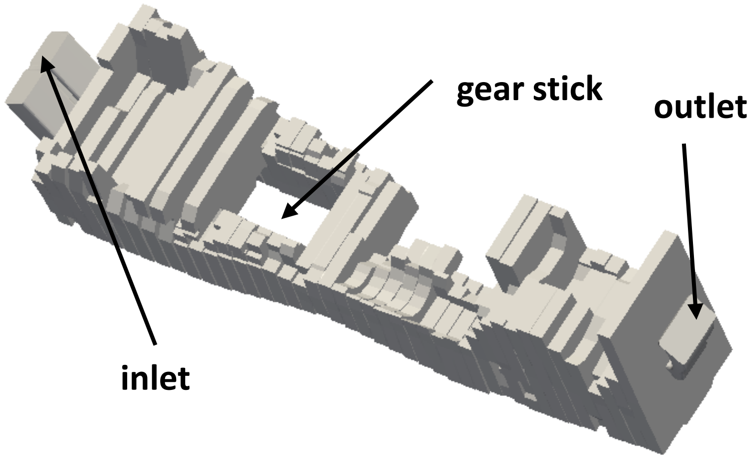


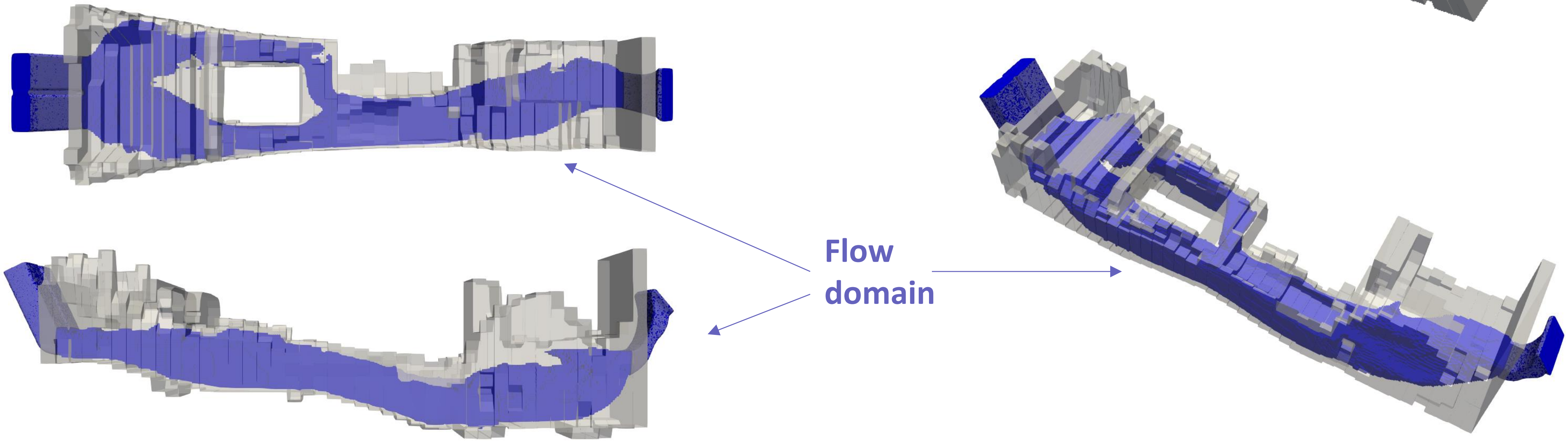
~45% J_{p_t} reduction

Papoutsis-Kiachagias et al. An Adjoint-based Topology Optimization Framework for Fluid Mechanics and Conjugate Heat Transfer in OpenFOAM. 8th OpenFOAM Conference, 2020.

HVAC duct 3D Aerodynamic TopO (1/4)

Re = 3000 (turbulent flow, Spalart-Allmaras model)
~5 Million design variables (excluding the inlet and outlet ducts)
<u>Objective:</u> Min. total pressure drop (J_{p_t})
<u>Constraint:</u> Fluid must occupy <70% of the design space



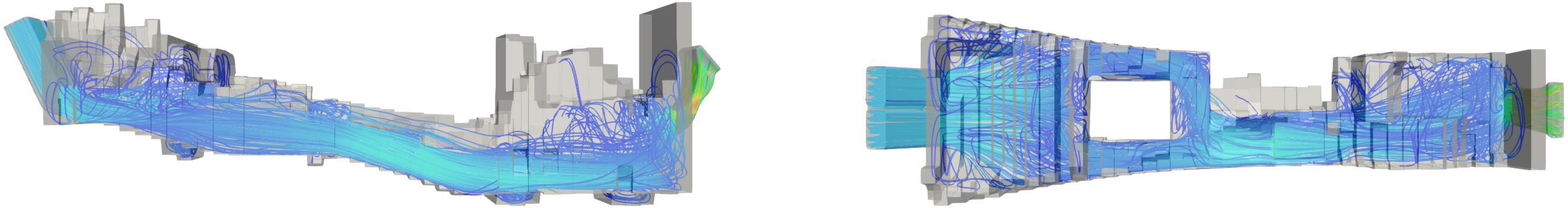


Volkswagen

Papoutsis-Kiachagias et al. An Adjoint-based Topology Optimization Framework for Fluid Mechanics and Conjugate Heat Transfer in OpenFOAM. 8th OpenFOAM Conference, 2020.

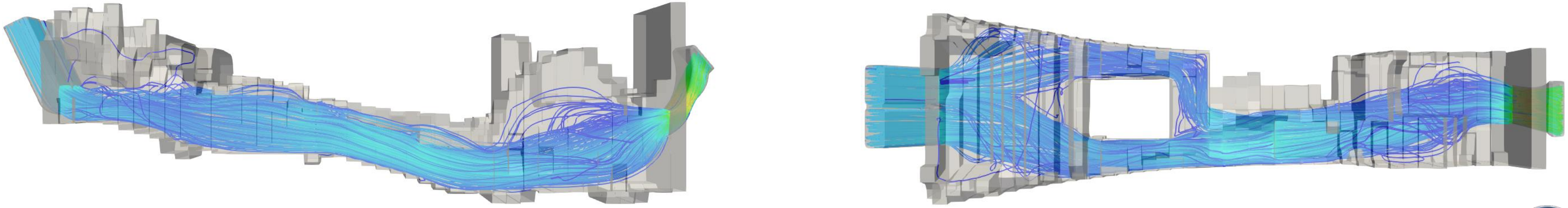
HVAC duct 3D Aerodynamic TopO (2/4)

Initial



~43% J_{p_t} reduction

Optimised



A large part of the secondary flows has been suppressed by TopO.

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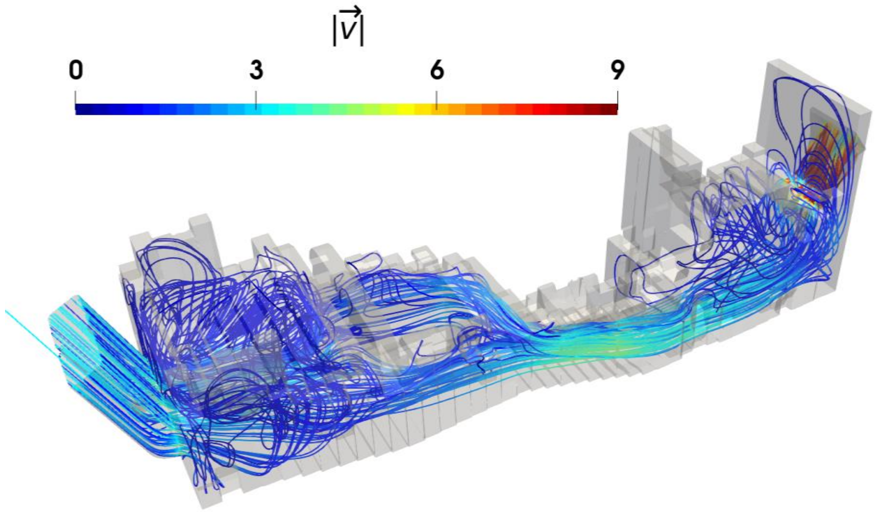
Papoutsis-Kiachagias et al. An Adjoint-based Topology Optimization Framework for Fluid Mechanics and Conjugate Heat Transfer in OpenFOAM. 8th OpenFOAM Conference, 2020.

HVAC duct 3D Aerodynamic TopO – Revisited using the TDDC Adjoint (3/4)

Turbulent case: Re=20000
Starting from an all-fluid domain.
Objective: min. total pressure drop
Constraint: Fluid volume < 50% of total volume

- ***denTopO*** **using the TDDC adjoint reduces the objective function by 32.7% compared to 30.3% when using the "standard" continuous adjoint discretisation!**



Volume-weighted Total Pressure Losses [Watt]

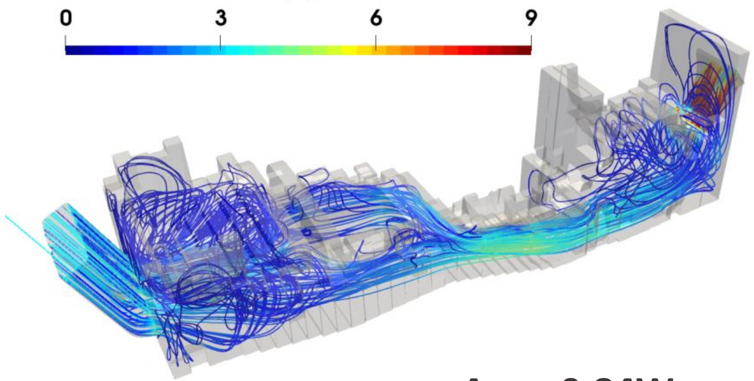
	Baseline	Std. Continuous Adjoint Optimised	TDDC Adjoint Optimised
denTopO flow solver	0.84	0.673	0.649
body-fitted grid solver	0.84	0.585	0.565

Volkswagen

Galanos N. “Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent “, PhD, NTUA, 2025.

HVAC duct 3D Aerodynamic TopO – Revisited using the TDDC Adjoint (4/4)

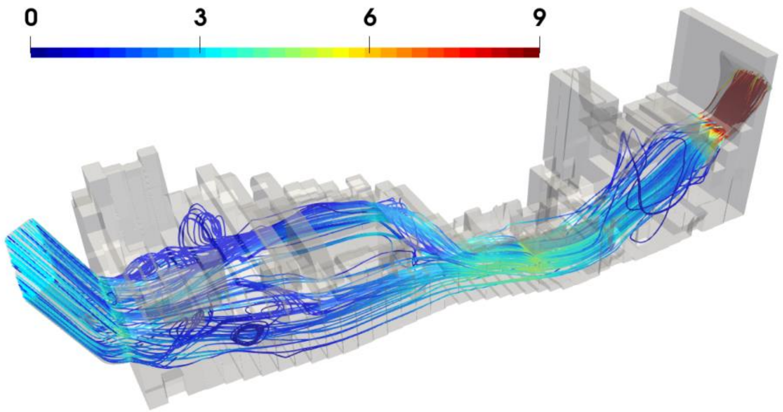
Baseline geometry



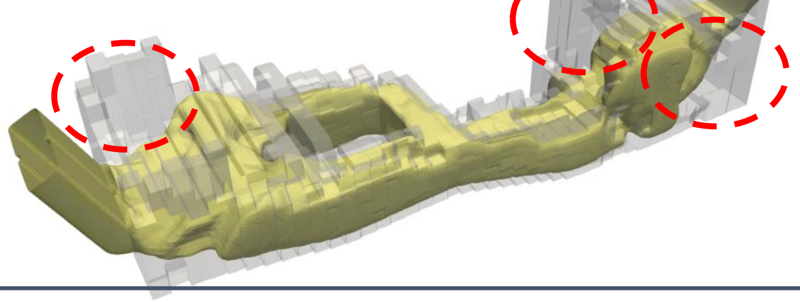
Δp_t = 0.84W

To reduce Δp_t *denTopO* solidifies areas of flow recirculation!

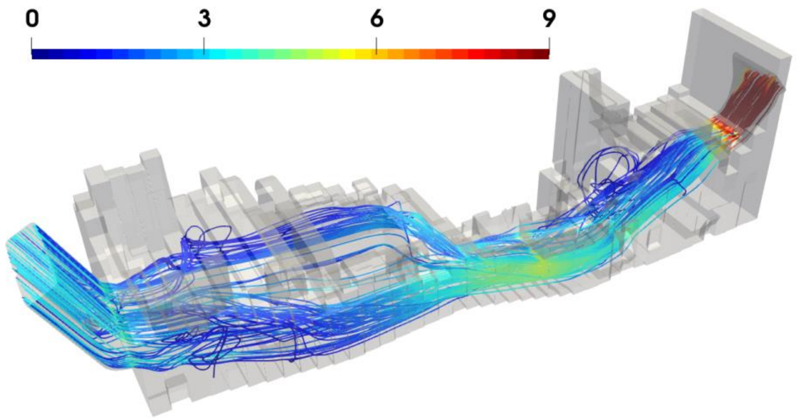
Optimised using the "standard" continuous adjoint solver



Δp_t = 0.585W
30.3% reduction



Optimised using the TDDC adjoint solver

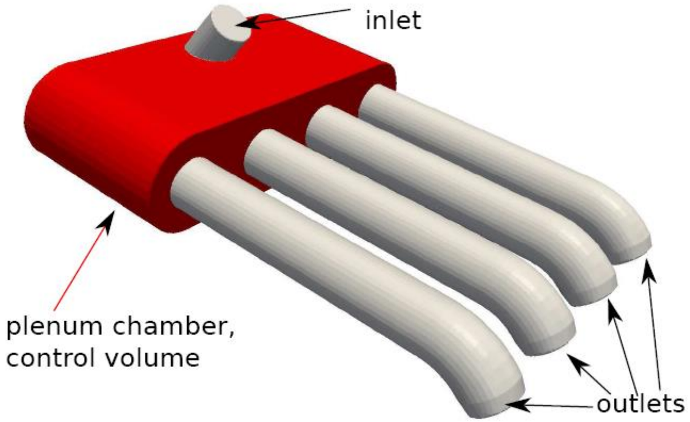


Δp_t = 0.565W
32.7% reduction

Volkswagen

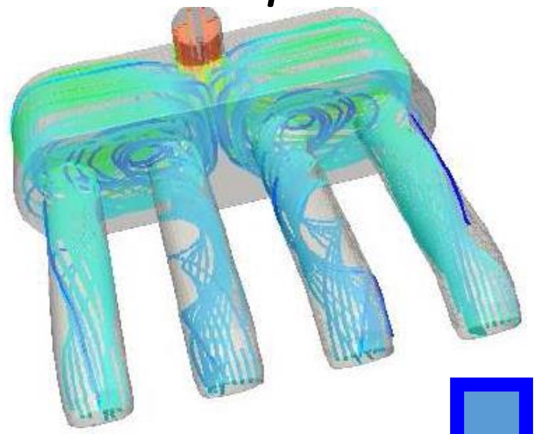
Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

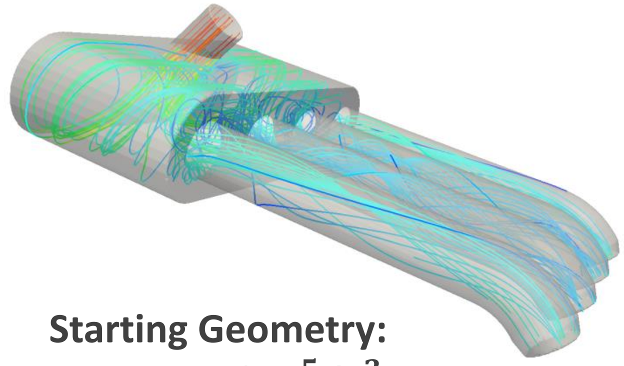
An Industrial Application of Mono-Fluid TopO



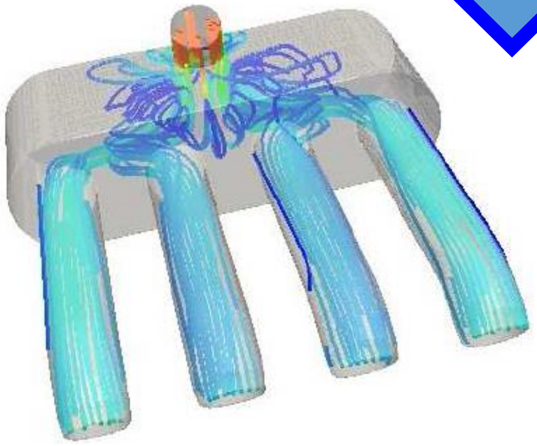
$$F_2 = \int_\Omega \frac{\nu+\nu_t}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)^2 d\Omega + \int_\Omega \alpha v_i^2 d\Omega$$

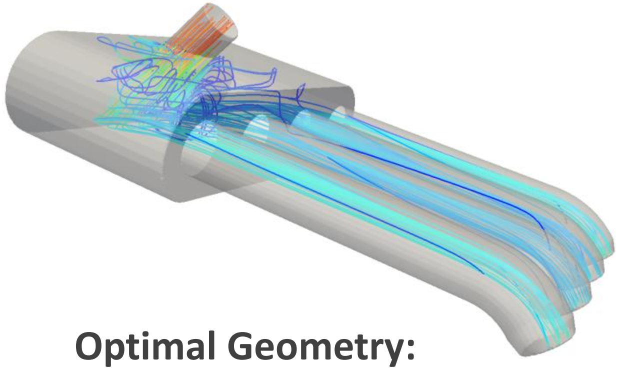
$$c_3 = \left[\frac{\int_\Omega\left(1-\frac{\alpha}{\alpha_{Max}}\right)d\Omega}{\Omega_{tot}} - r_{tar}\right]^2 = 0$$





Starting Geometry:
$F = 0.25\ m^5/s^3$



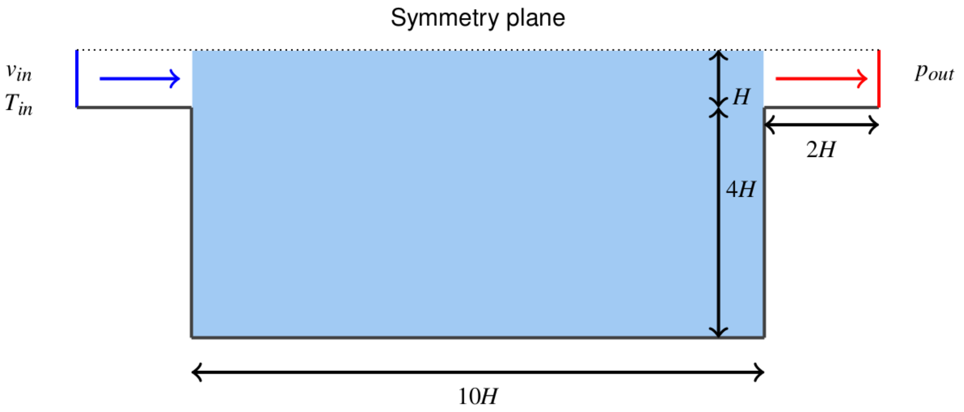


Optimal Geometry:
$F = 0.177\ m^5/s^3$

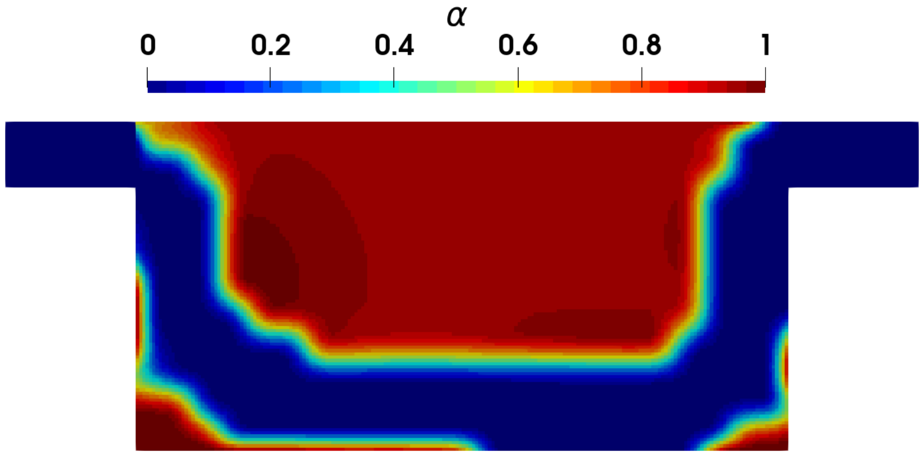
TopO of the plenum chamber of a student racing car, targeting min. fluid power dissipation , by additionally using a fluid volume constraint (=50% of the overall volume Ω). The optimal design yields a 29% reduction in the obj. function value.

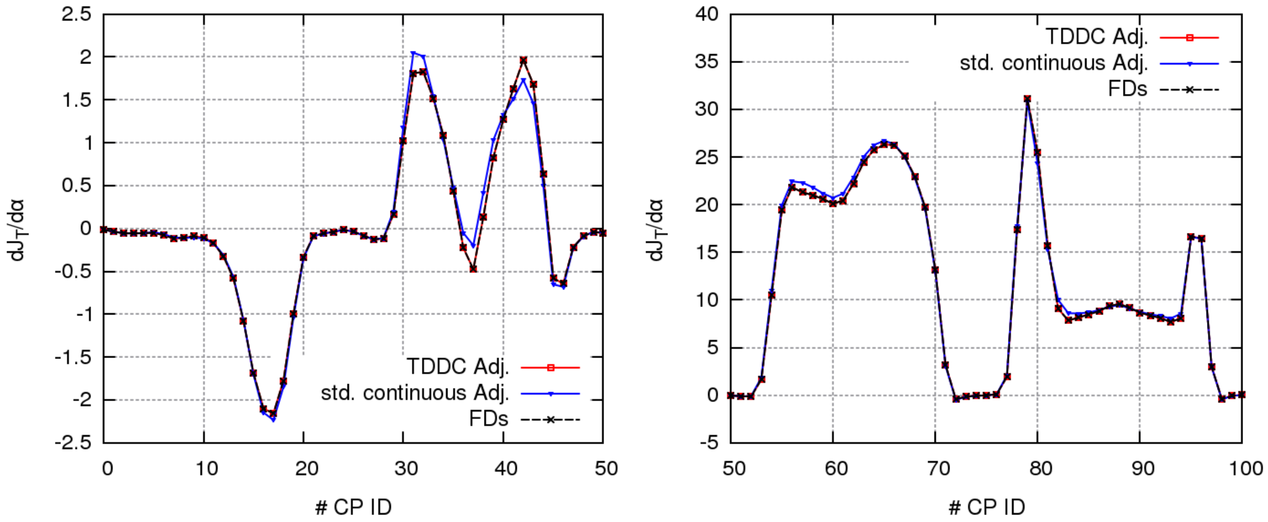
denTopO for the Cooling of a 2D Heat Sink – The TDDC Adjoint (1/2)

Turbulent case: Re=2500
 # Cells = 21.6K, H=0.1m
The solid domain is heated by applying Q=1kW/m²
Evaluation of the TDDC Adjoint!



A duct was created by just giving appropriate α values to the control points of the morphing lattice





Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

denTopO for the Cooling of a 2D Heat Sink – Parametric Studies (2/2)

- **Objective**: min. average T over the solid.
- **Constraints**:
  - volume-weighted Δp_t < 0.435mW.
  - Fluid volume < 49% of total volume.

Body-fitted *CHT* evaluation

	Δp_t [mW]	T [°C]
Setup C1	0.259	123.3
Setup C2	0.293	173.9
Setup C3	0.288	157.7

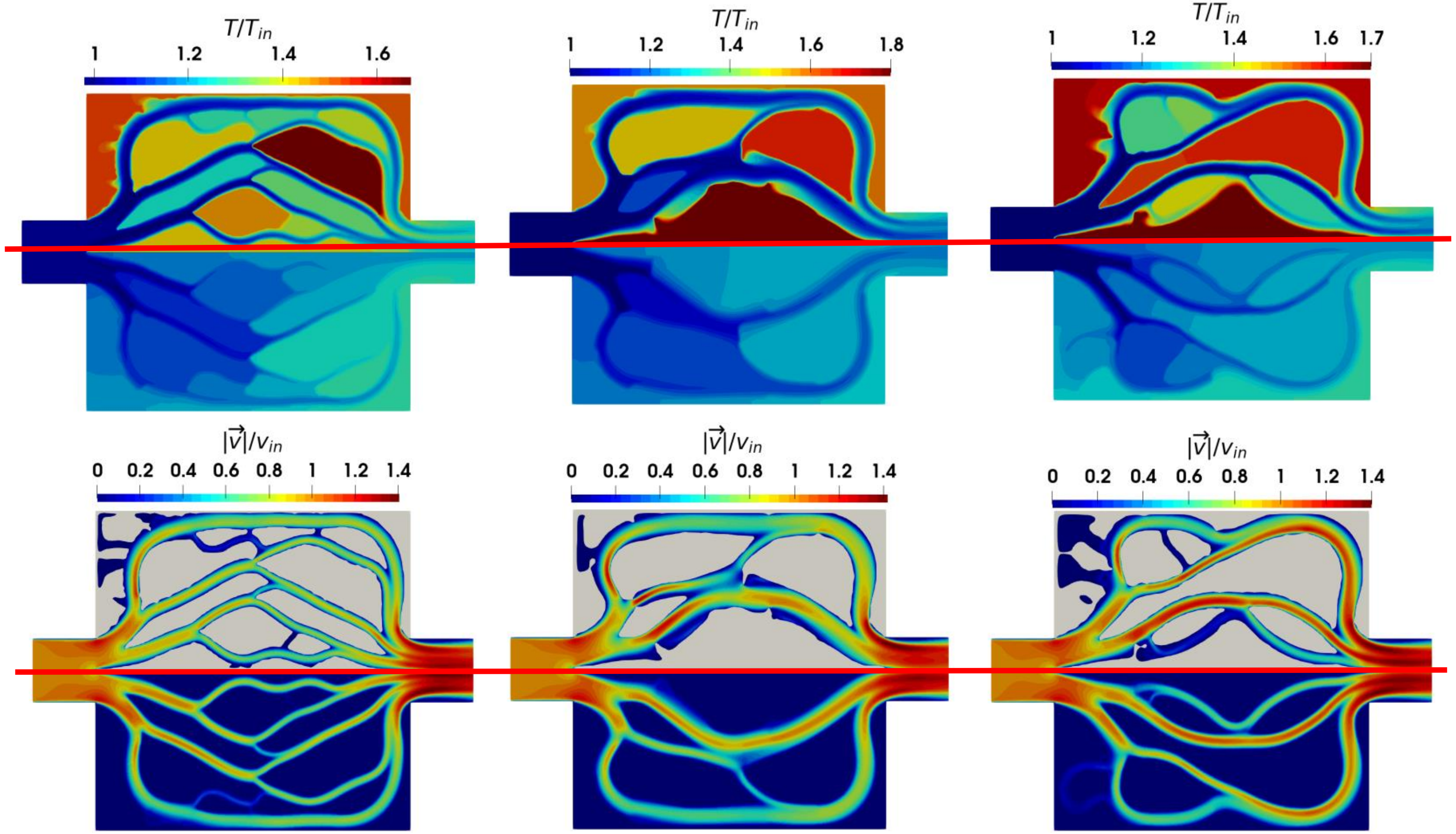
***denTopO* flow solver**

	Δp_t [mW]	T [°C]
Setup C1	0.435	50.2
Setup C2	0.435	61.2
Setup C3	0.435	58.9

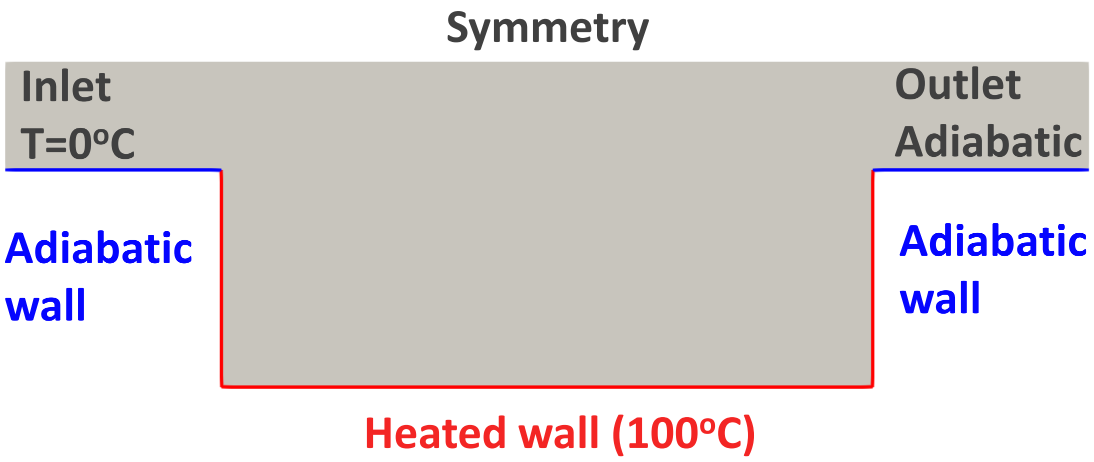
Filtering parameter

Projection parameter

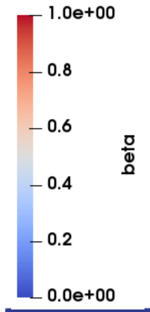
Setup C1 (r=H/4, b=10) Setup C2 (r=H/2, b=10) Setup C3 (r=H/2, b=20)



A Mono-Fluid CHT TopO Case – Pareto Front (1/2)

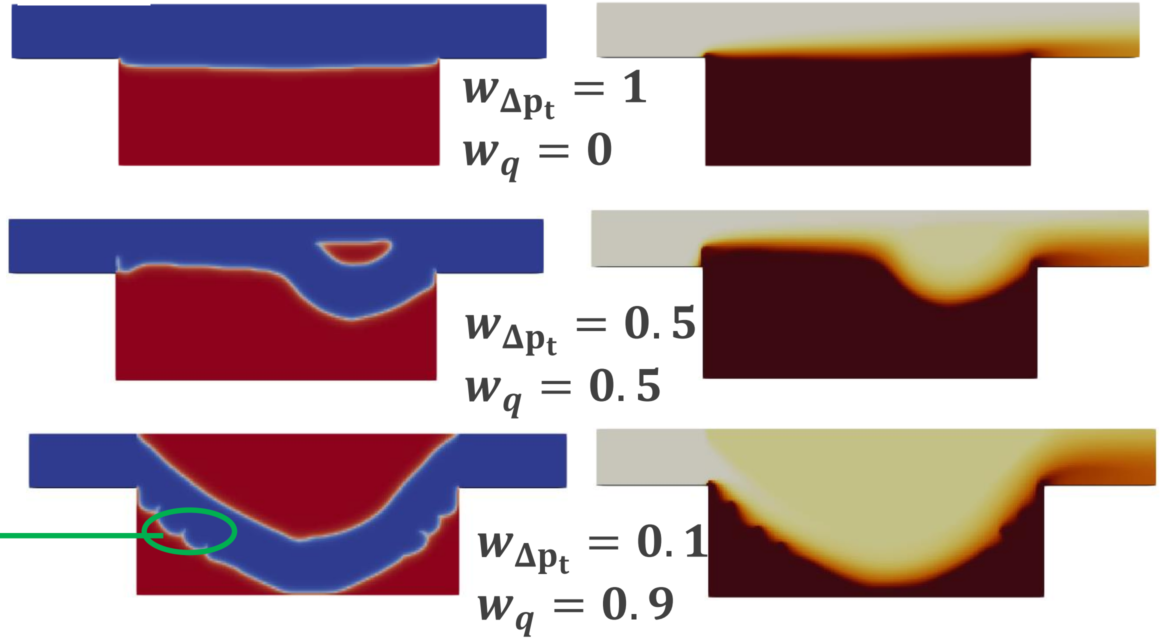


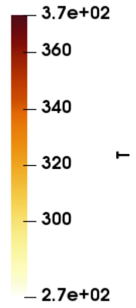
- Min. total pressure losses ($J_{\Delta p_t}$).
- Max. heat flux difference between the inlet and the outlet (J_q).
- Constraint: Fluid must occupy less than 50% of the design space.
- Re = 166 (laminar flow).



Running the code several times with different weights may lead to a front of non-dominated solutions

$$J = w_{\Delta p_t} J_{\Delta p_t} + w_q J_q$$

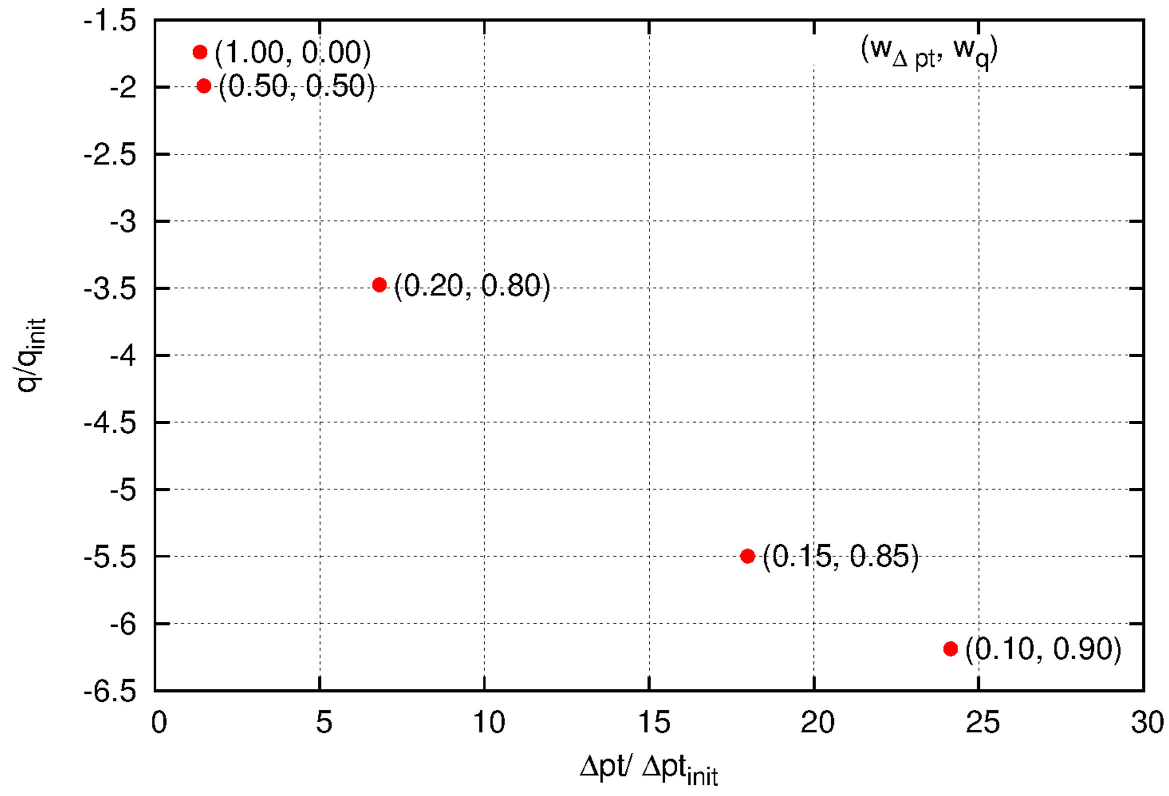




Fins formed to
enhance heat transfer

Papoutsis-Kiachagias et al. An Adjoint-based Topology Optimization Framework for Fluid Mechanics and Conjugate Heat Transfer in OpenFOAM. 8th OpenFOAM Conference, 2020.

A Mono-Fluid CHT TopO Case – Towards the Pareto Front (2/2)



$$J = w_{\Delta p_t} J_{\Delta p_t} + w_q J_q$$

Running the code several times with different weights may lead to a front of non-dominated solutions

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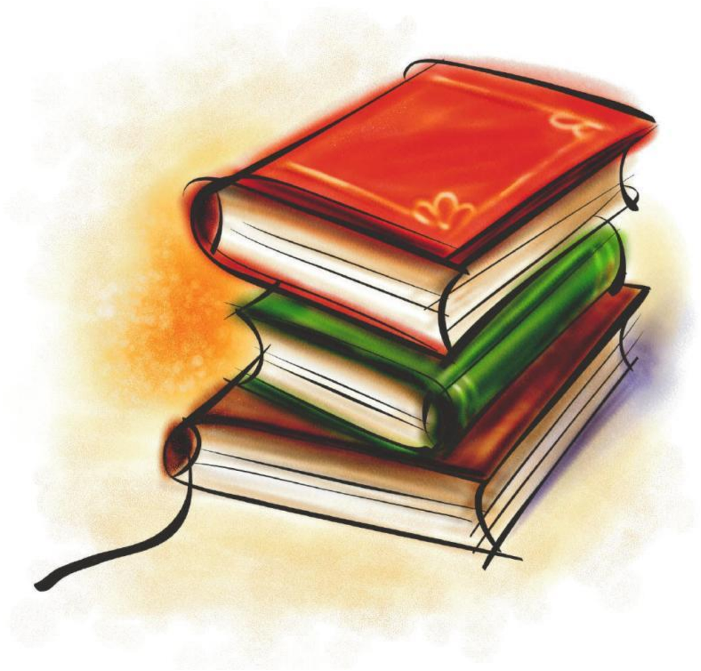
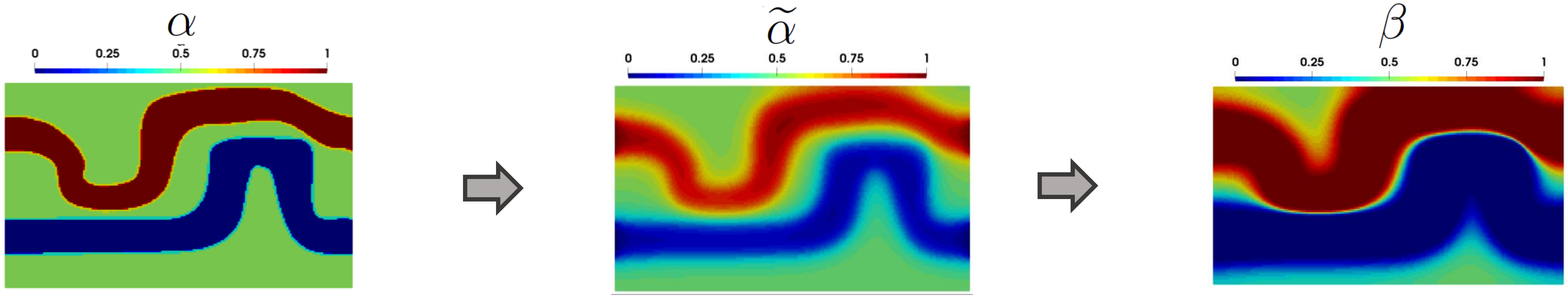


denTopO for Bi-fluid Heat Exchangers (HEx)

Bi-Fluid CHT denTopO for HEx design

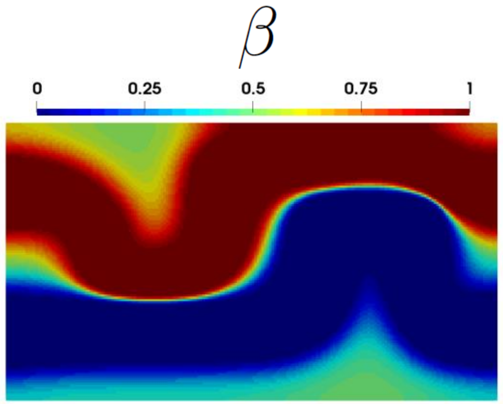
A single field of design variables (impermeability α) is used to describe both fluid and solid domains. Not enough if more than one fluids!



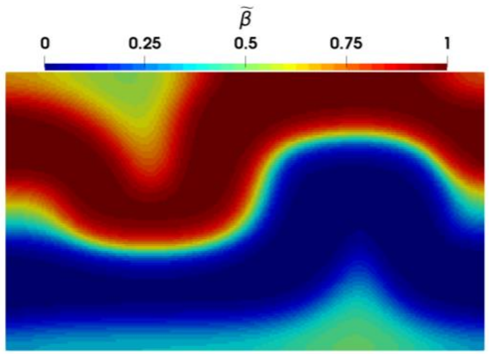
Bi-Fluid CHT denTopO for HEx design

Double Helmoltz-PDE filtering and projection to ensure the separation of the two fluids and control the min. solid thickness between them.

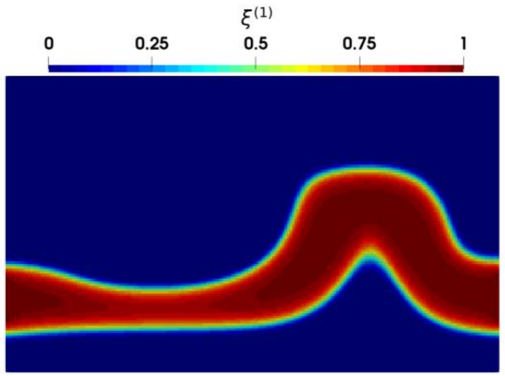
Solid domain: $\xi^{(1)}, \xi^{(2)} \sim 0$



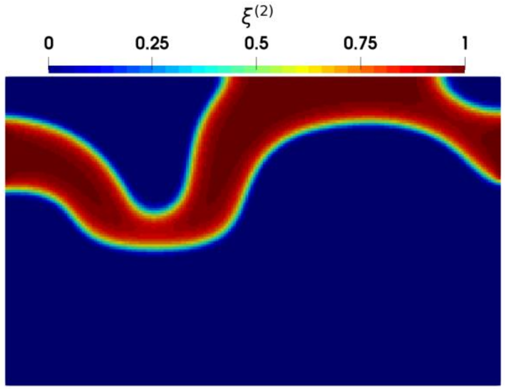
Second filtering



Second projection with a low and high threshold



$\xi^{(1)} \sim 1, \xi^{(2)} \sim 0$:
Fluid 1 domain

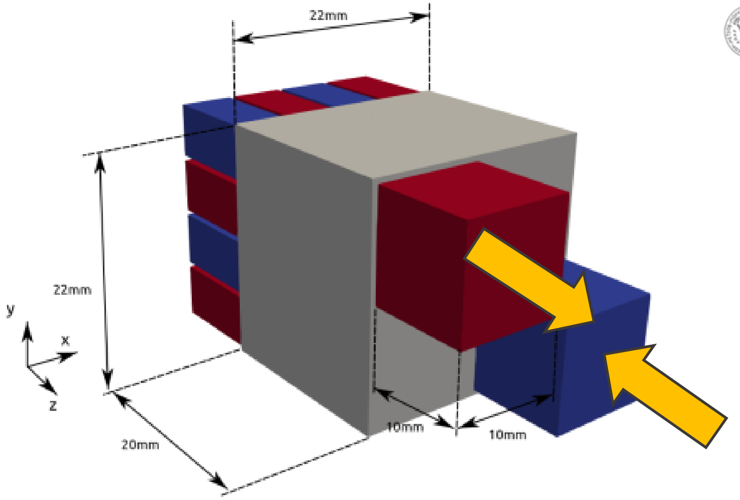


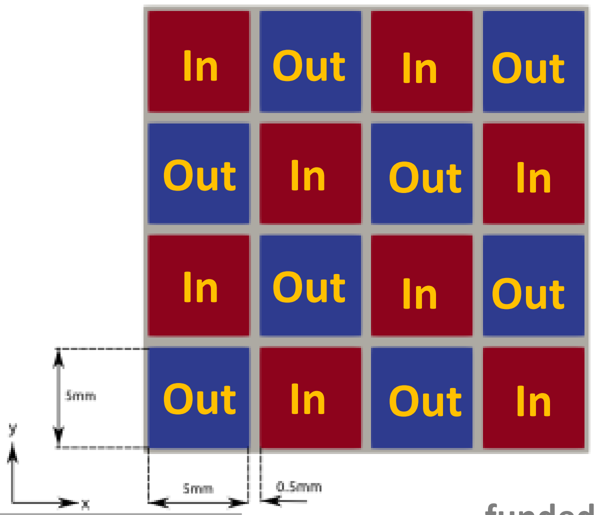
$\xi^{(2)} \sim 1, \xi^{(1)} \sim 0$:
Fluid 2 domain

L.C. Hoghoj, D.R. Norhave, J. Alexandersen, O. Sigmund, C.S. Andreasen. "Topology Optimization of Two Fluid Heat Exchangers". Int. J. of Heat and Mass Transfer (2020).

Case D - denTopO of a Bi-Fluid HEx with multiple Inlets/Outlets

- ❑ Grid: ~1.1M cells.
- ❑ The objective function includes the:
  - Outgoing heat of the hot fluid at its exit (min.)
  - Volume-weighted total pressure drop of each fluid (min.).
- ❑ Constraints:
  - Equi-distribution of the cold fluid at its exits.
  - Fluid & solid should be separated by at least 0.5mm of solid material (~two grid cells).
  - Volume of each fluid < 35% of the total *HEx* volume
- ❑ Physical properties:
  - Fluid 1: water
  - Fluid 2: air
  - Solid: aluminum (k=136 W/m.K)





Structural and Multidisciplinary Optimization, 65(9), 245, 2022.

funded by MITSUBISHI HEAVY INDUSTRIES

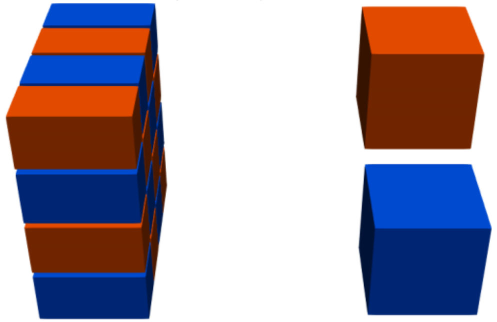
Stage 1: Initialisation of α using a low-fidelity model for denTopO

- **Start from a completely solidified domain. Replace the RANS eqs. (Hi-Fi) with a simplified Darcy-based (Low-Fi) model, easily carving the initially solid design space (iniTopO). Wall clock time: ~1hour on 4 AMD EPYC 7452 32-core CPUs (128 cores) (100 Opt. Cycles).**

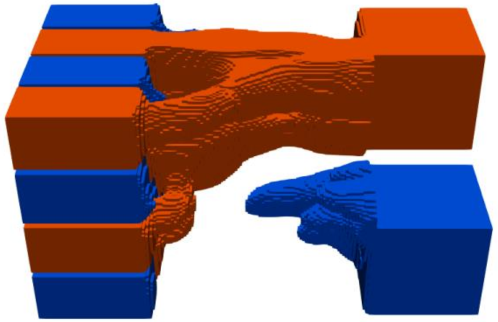
The Darcy flow model:

$$v_i^{(\gamma)} = -\frac{1}{\beta_{max}\left(Da + I\left(\xi^{(\gamma)}\right)\right)}\frac{\partial p^{(\gamma)}}{\partial x_i}$$

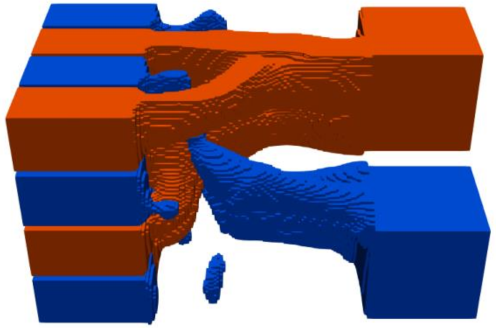
Opt. Cycle 1



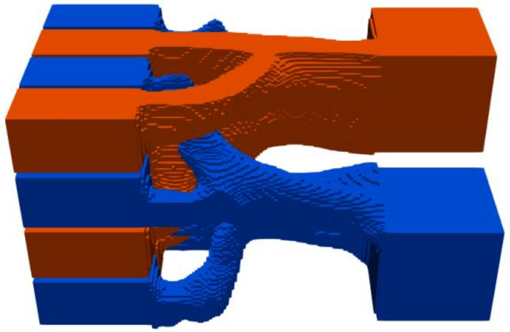
Opt. Cycle 5



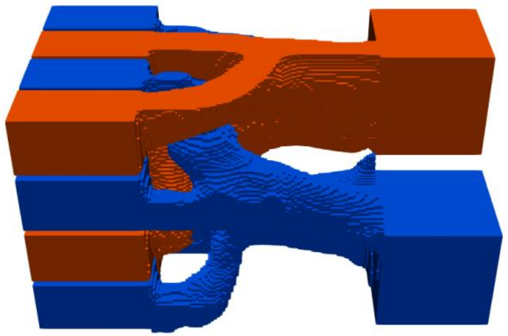
Opt. Cycle 10



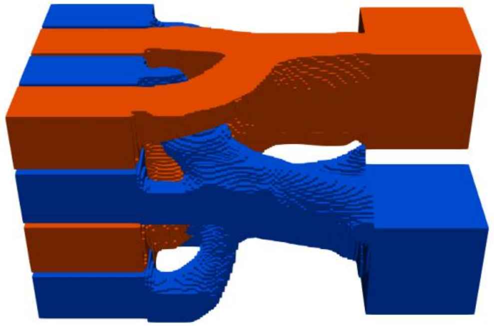
Opt. Cycle 20

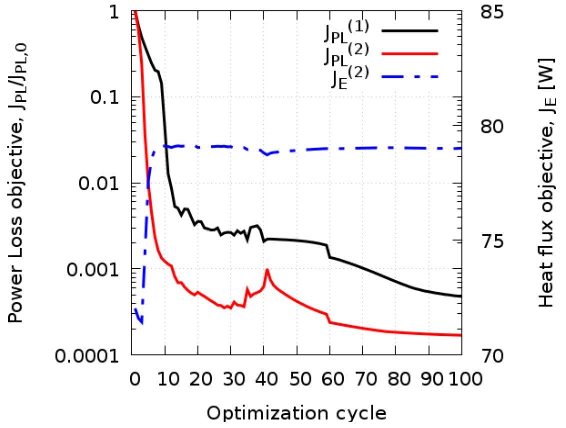


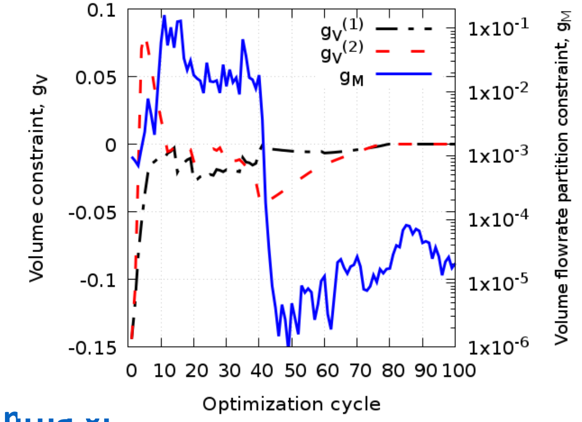
Opt. Cycle 50



Opt. Cycle 100

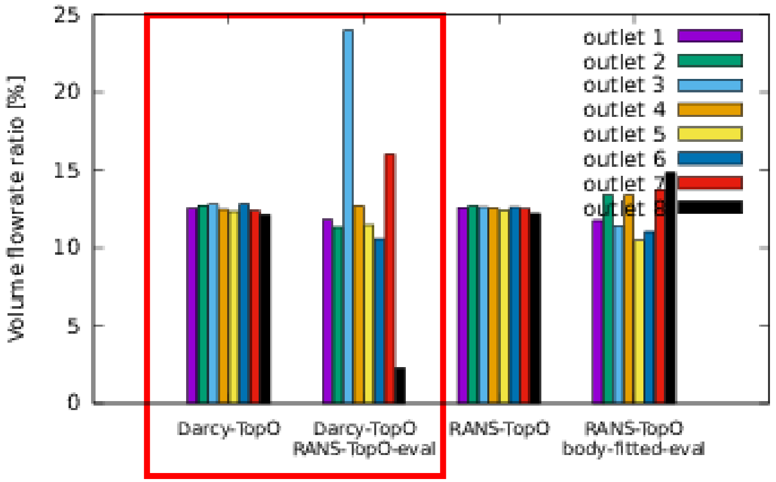


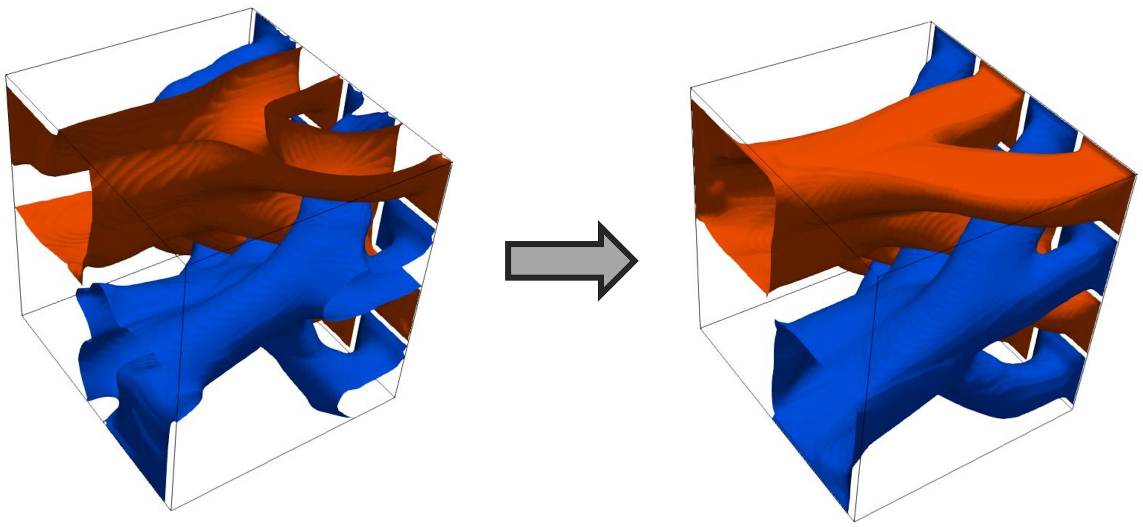


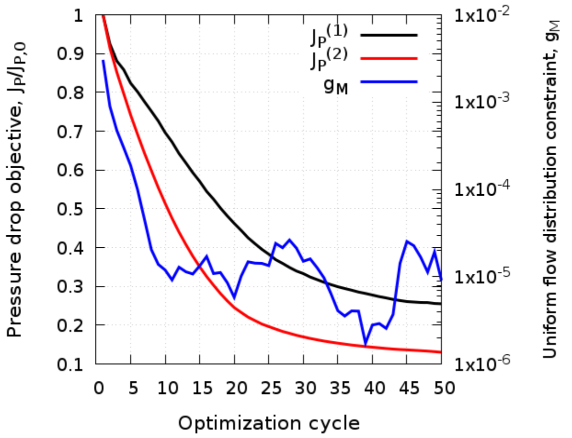


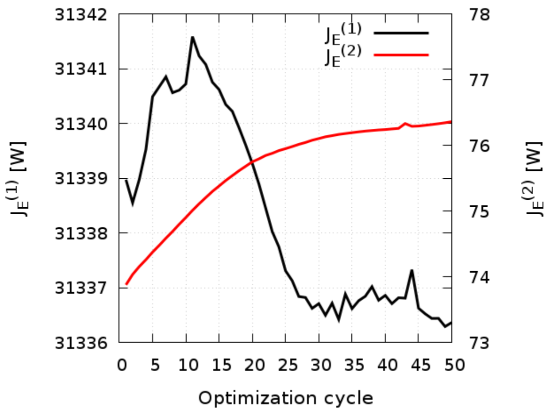
Stage 2: RANS-based denTopO

By evaluating the previous solution on the Hi-Fi RANS tool, the flow equi-distribution constraint is not met!
A Hi-Fi RANS *denTopO* follows, starting from the outcome of the Darcy-*denTopO* (*iniTopO*).
Cost: ~9.3 hours on 128 CPU cores (50 Opt. Cycles)







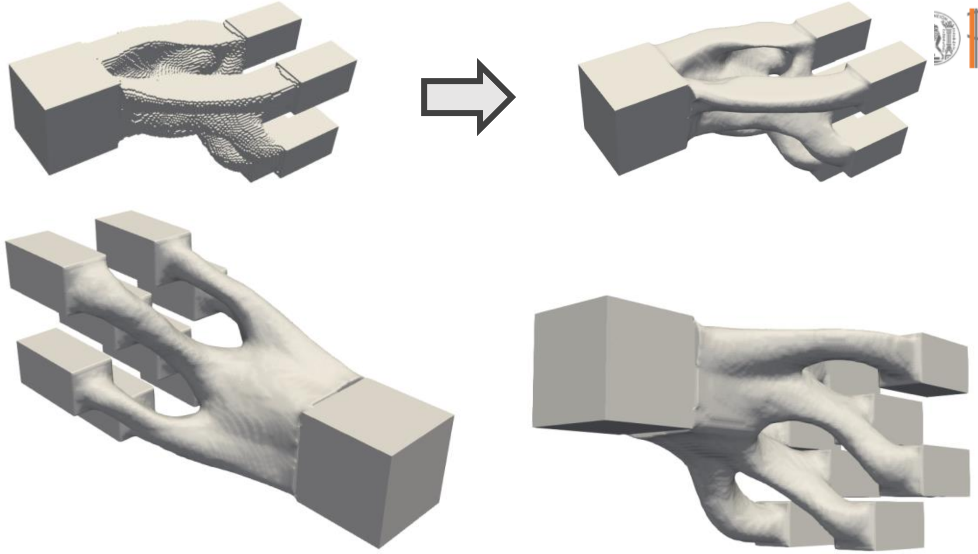


From the Darcy-denTopO to the RANS-denTopO solution.

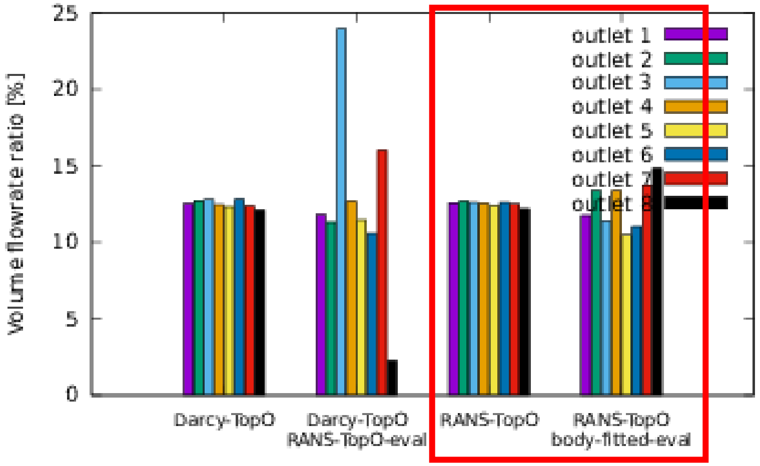
RANS-based denTopO solution re-evaluation on Body-Fitted grids

- **Adequately stretched grids close to the FSI are generated for the two fluids (~2.1M cells) and the solid (~3M cells).**

- ***Cold fluid not equi-distributed to its many outlets!***

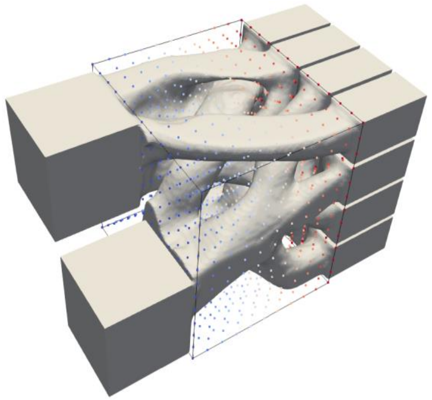


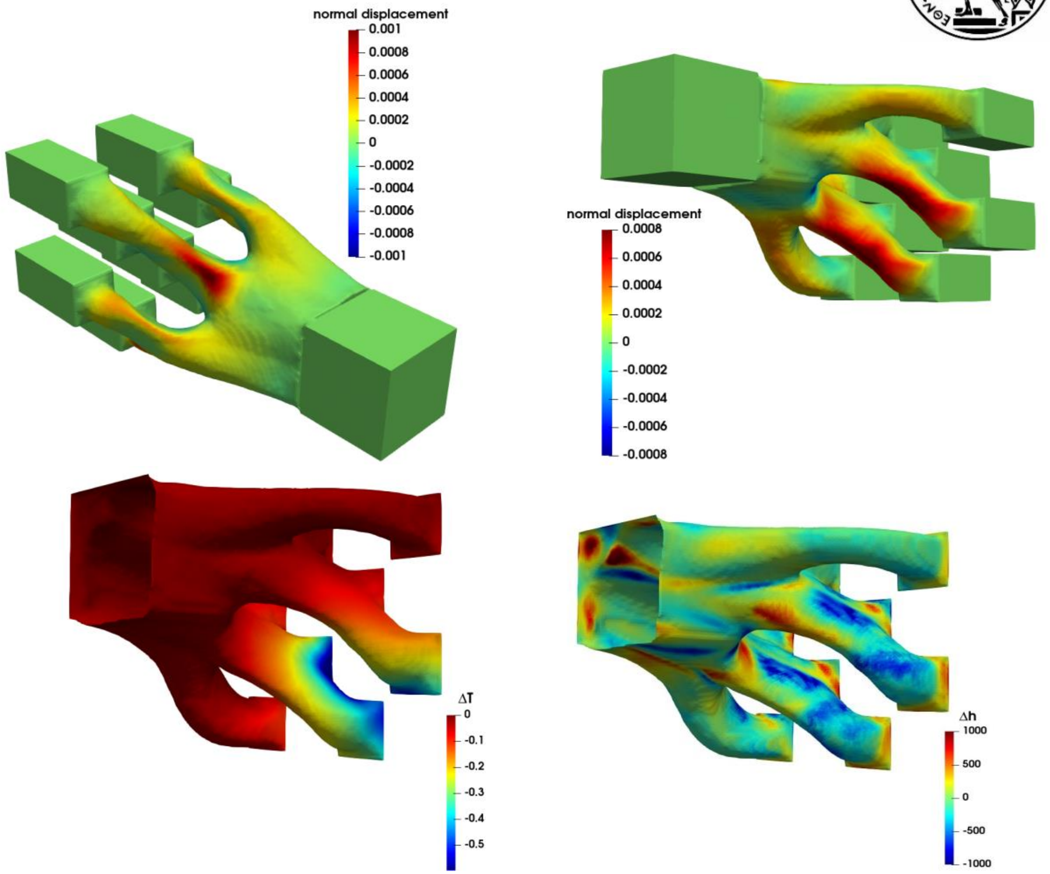
Outcome of the TtoST for the cold (left) and hot (right) fluid



Stage 3: CHT Bi-Fluid ShpO

Volumetric B-Splines morphing of the design space (a single lattice of 12x12x11 control points).





ShpO run based on the software developed in the PhD thesis of K.T. Gkaragkounis.

Cost: ~15 hours on 128 cores

35% reduction in Δp_t for the cold fluid and 25% reduction for the hot fluid, without damaging the cooling performance!

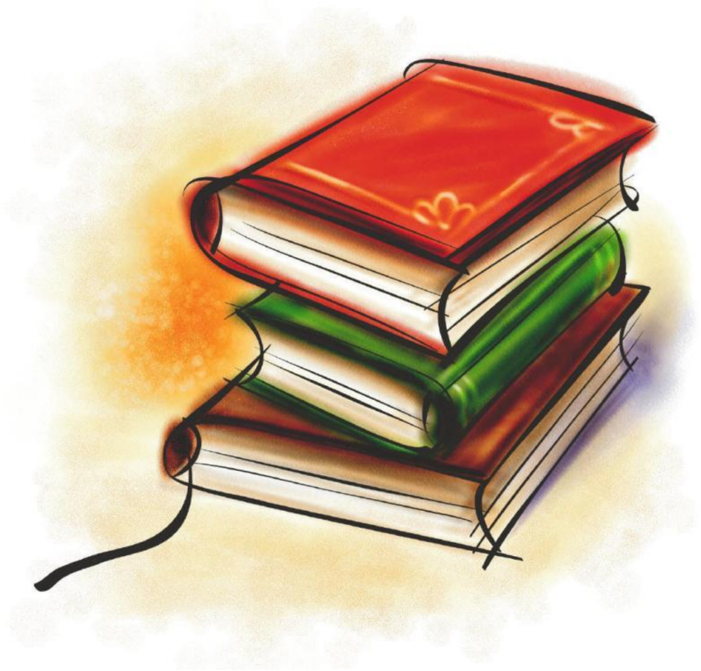
<u>Overall CPU Cost</u>: ~1 day on 128 cores for the three-stage design!

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

Lessons Learned

Low-Fi Flow Models are extremely useful in TopO!
Starting from an all-solid initialisation, *denTopO* computes a full *HEx* connecting inlets with outlets!
***ShpO* can be used synergistically with *denTopO* to eliminate its inaccuracy though at higher computational cost!**

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.



- **The *cut-cell method* for TopO (*cutCellTopO*) of turbulent flow problems with CHT**

Why cut-cells in TopO?

✗ **Pitfalls in the "*standard*" *denTopO* approach:**

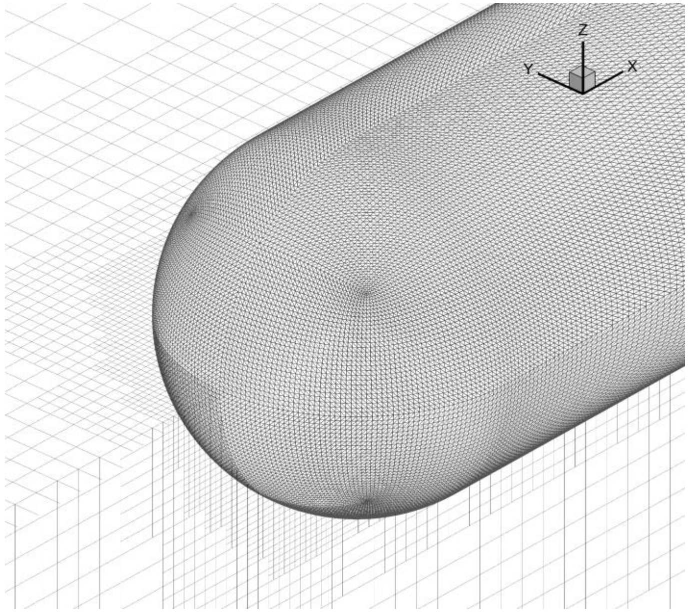
- **The Fluid Solid Interface (FSI) is not considered during *denTopO*.**
- **FSI conditions are only imposed in a weak sense through the usage of Brinkman penalisation terms.**
- **The lack of FSI conditions may also allow some spurious flow leakage inside the solid.**

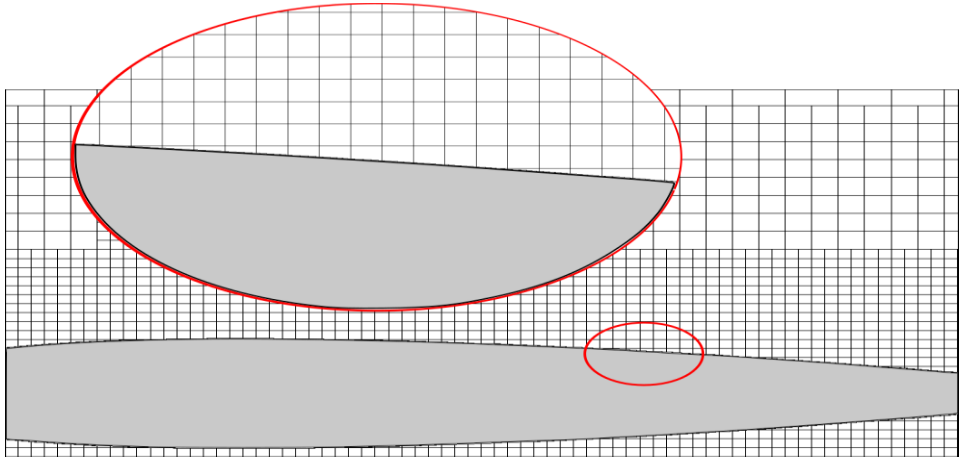
✗ **When re-evaluating the outcome of *denTopO* on a body-fitted grid, different performances are usually computed!**

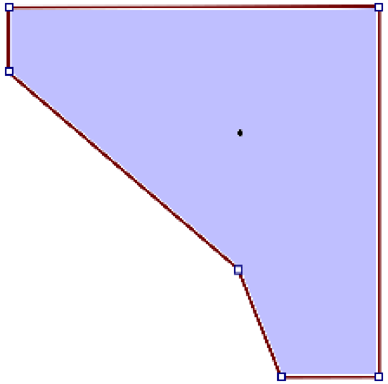
✓ **To improve the accuracy of the flow solver during TopO, this work makes use of the *cut-cell method* which computes the FSIs and applies accurate conditions there!**

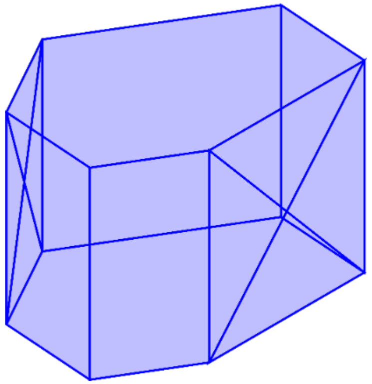
The Cut-Cell method

- **Immersed Boundary Methods - ‘Body fitted’ Cartesian mesh**
- **Intersections with Geometry; Generation of Cut-cells**
- **Efficient and automated mesh generation**
    - $1\ min$ **per million cells**
- **Hierarchical Data Structure**
    - **Reduced storage requirement**
    - **Isotropic cell refinement**

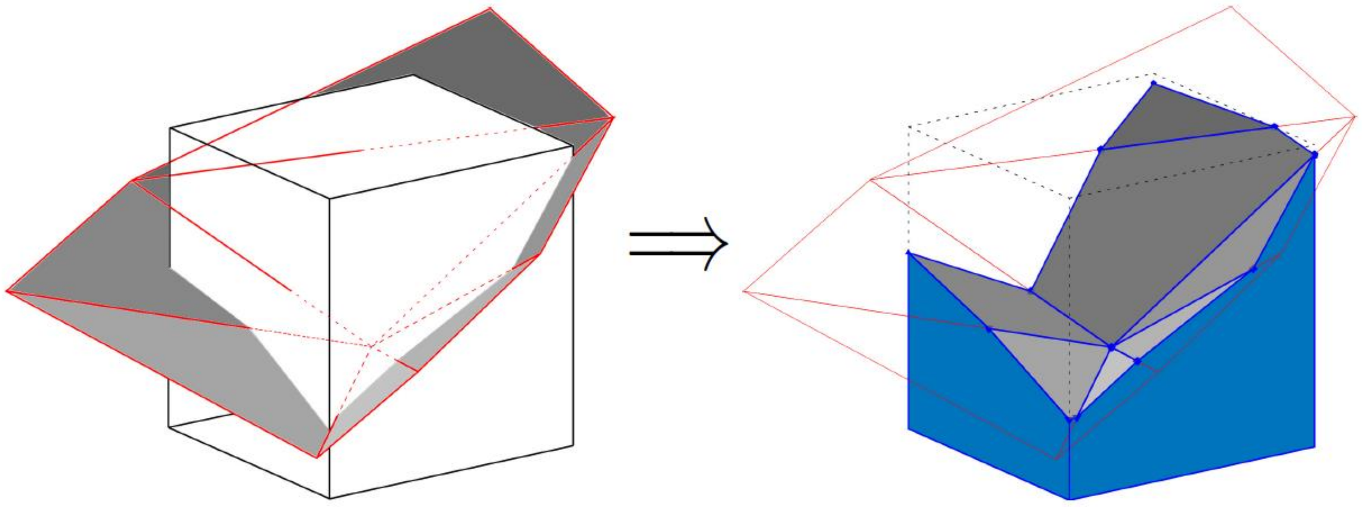








The Cut-Cell method



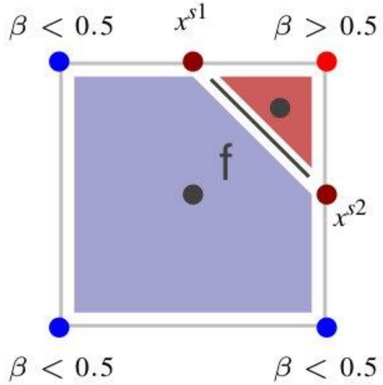
The cell–cutting algorithm follows a two–step procedure:

(A) Each intersecting triangle is clipped inside the Cartesian cell. To do so, the Sutherland–Hodgman algorithm is used to compute the new polygon that is entirely encompassed within the Cartesian cell.

(B) The cut-cell is created by identifying the solid and fluid parts of the Cartesian cell and fluid and solid faces are identified by connecting the computed intersection points with the Cartesian cell edge points.

The cutCellTopO method

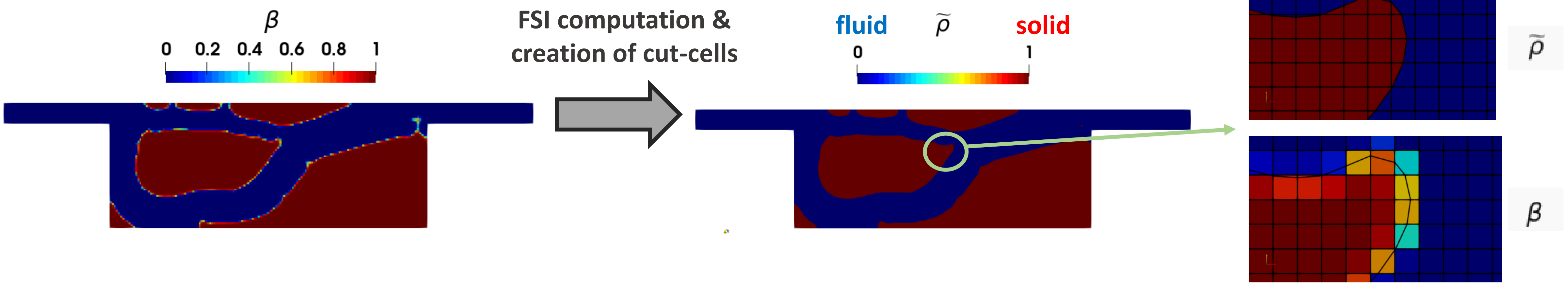
$$x_i^s = x_i^A + \frac{0.5 - \beta^A}{\beta^B - \beta^A}\left(x_i^B - x_i^A\right)$$



Same design variables as in the "*standard*" *denTopO* method with the same filtering and projection methods!

- **In each optimisation cycle, the FSI is computed (and used) as:**
  - **The β=0.5 iso-surface in mono-fluid *cutCellTopO***
  - **The $\xi^{(1)}+\xi^{(2)}$=0.5 iso-surface in bi-fluid *cutCellTopO***

The method supports *Adaptive Mesh Refinement* (*AMR*) close to the FSI using the β (in mono-fluid) or the $\xi^{(1)}+\xi^{(2)}$ (in bi-fluid) fields as indicators.
Programmed in *OpenFOAM* using its *dynamicFvMesh* functionality to perform changes in grid topology!

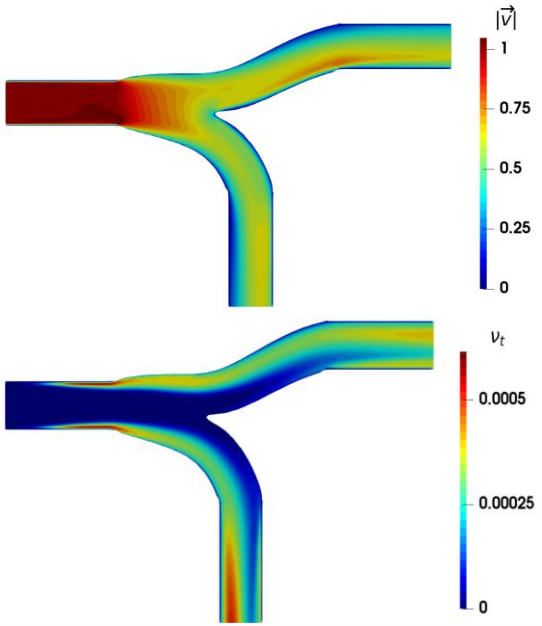


Case Revisited – cutCellTopO of a 1-Inlet-2-Outlet Square Box

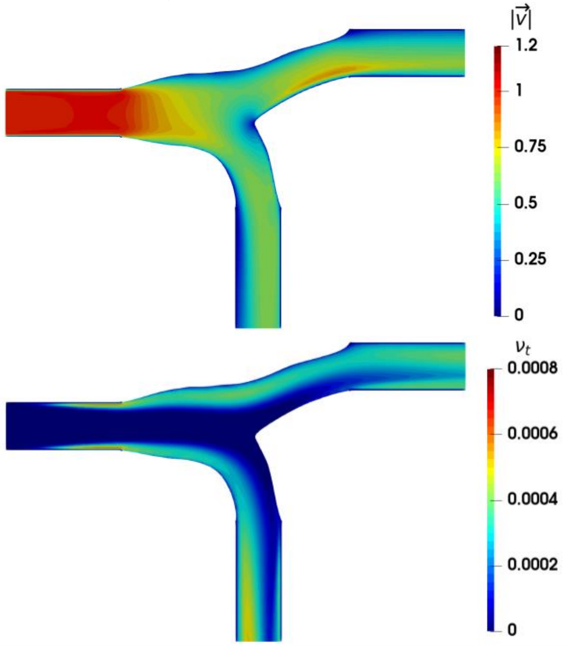
Objective: min. total pressure drop
Constraint: Fluid volume < 46.2% of total volume

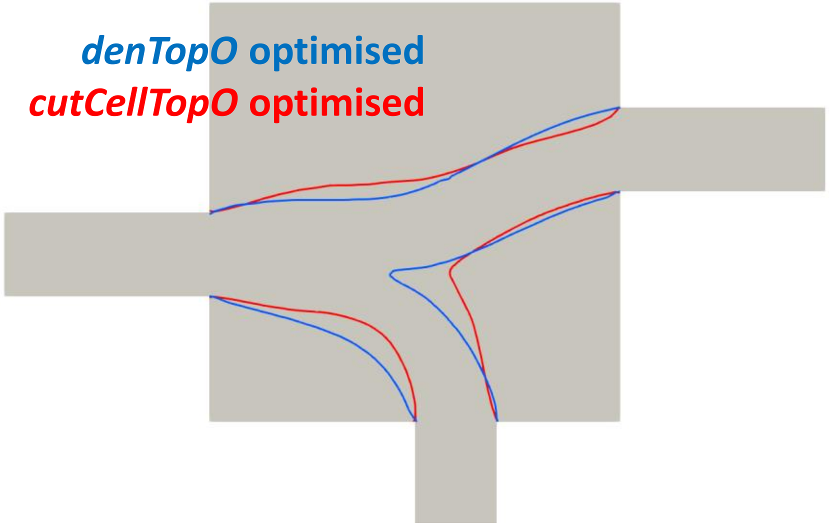
***cutCellTopO* leads to a better solution as it accurately computes J in each cycle**

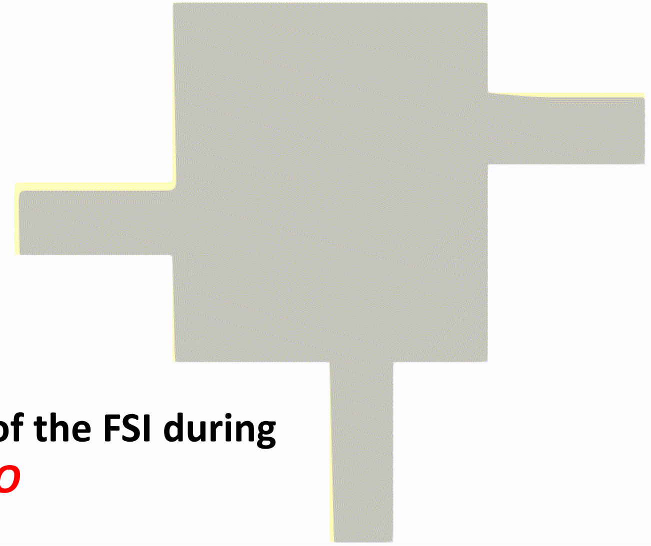
Optimal *denTopO* re-evaluated
Δp_t = 0.98mW



Optimal *cutCellTopO*
Δp_t = 0.93mW







Evolution of the FSI during *cutCellTopO*

Case Revisited - Adaptive Mesh Refinement (AMR) Strategy (1/2)

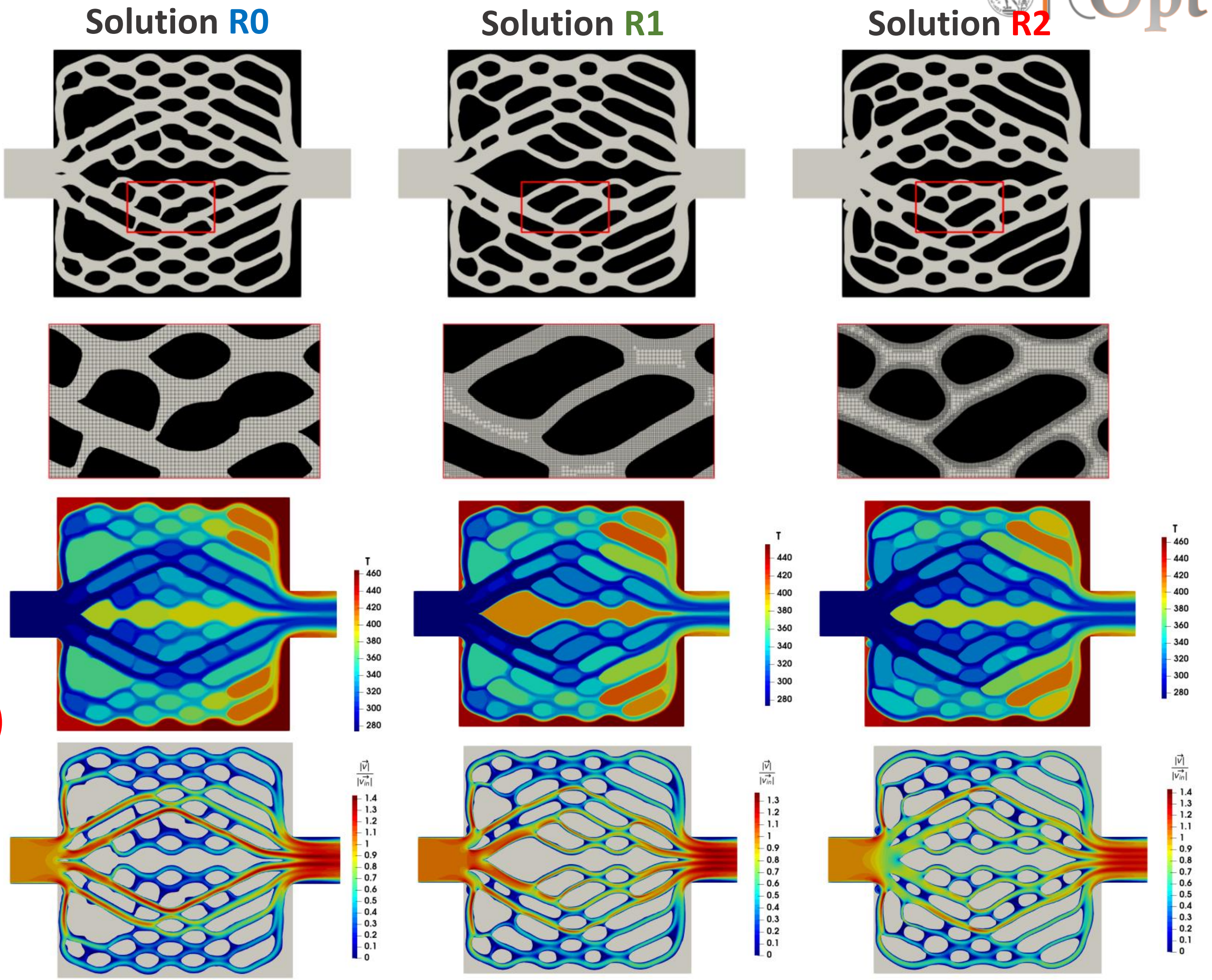
Re = 2500

Objective: min. average T over the solid

Constraints:

- $\Delta p_t <$ 0.31mW.
- Fluid volume < 49% of total volume

	Δp_t [mW]	Avg. T in solid [°C]
Solution R0	0.31	105.0
Solution R1	0.31	105.4
Solution R2	0.31	105.6



cutCellTopO with Adaptive Mesh Refinement (AMR)

Solution R0: Without AMR

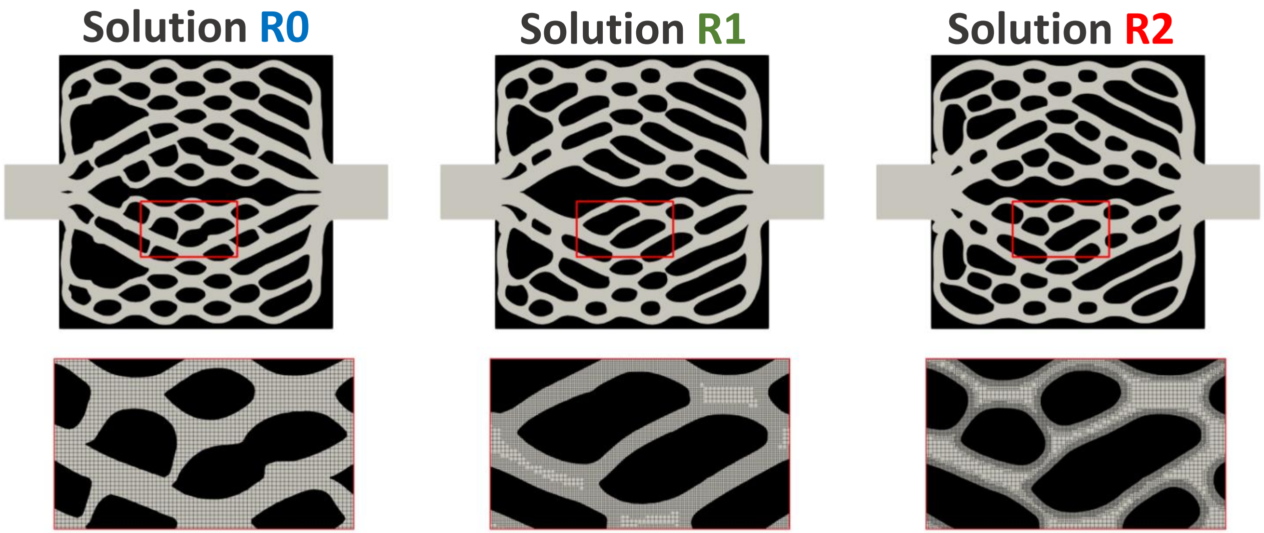
Solution R1: Single AMR

Solution R2: Double AMR

All three solutions respect the constraints

Case Revisited - Adaptive Mesh Refinement (AMR) Strategy (1/2)

	Δp_t [mW]		Avg. T in solid [°C]	
Solution R0	0.35	✗	114.6	✗
Solution R1	0.32	✗	107.4	✗
Solution R2	0.31	✓	105.6	✓

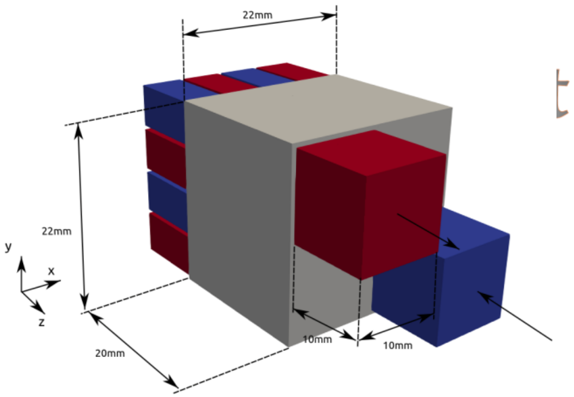


Solutions R0 and R1, re-evaluated on finer grids, give worse performance while failing to meet the pressure drop constraint!

Solution R0: Without AMR
Solution R1: Single AMR
Solution R2: Double AMR

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

Case Revisited – cutCellTopO of a multiple Inlet/Outlet HEx

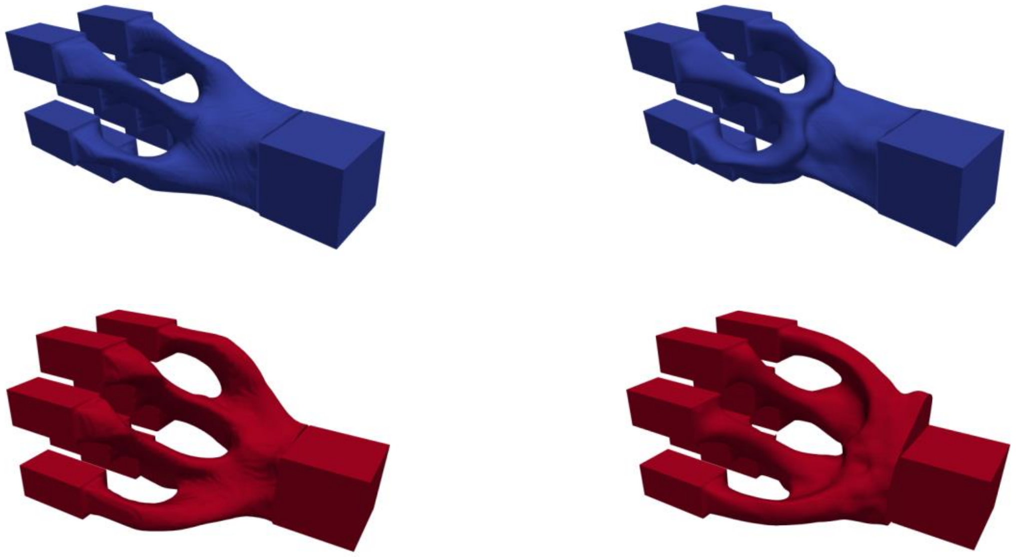


Objective: min. T of the hot fluid at the exit.
Pressure-drop constraints: $\Delta p_t^{(1)} < 1.18$mW, $\Delta p_t^{(2)} < 1.04$mW
Flow equi-distribution constraint: $g_M < 10^{-5}$
Initialisation: The Darcy *denTopO* optimised solution.

***denTopO* optimised** ***cutCellTopO* optimised**

Cold fluid

Hot fluid



Solution	$\Delta p_t^{(1)}$ [mW] (< 1.18)	$\Delta p_t^{(2)}$[mW] (<1.04)	g_M [x10^{-5}] (<1)	$T^{(2)}$ [°C]
Initial	1.83	6.09	824.3	52.77
denTopO	0.67	0.64	~1	48.88
denTopO re-eval	0.64	0.60	1.84 (>1)	53.30
cutCellTopO	1.13	1.05	0.85	52.78

The *cutCellTopO* solution satisfies all imposed constraints and has a lower $T^{(2)}$ at the hot fluid's exit.

Galanos N. "Topology and Shape Optimization in Fluid Mechanics & Conjugate Heat Transfer using Continuous Adjoint with Consistent ", PhD, NTUA, 2025.

Read more in:

- N. Galanos, E.M. Papoutsis-Kiachagias, K.C. Giannakoglou, Y. Kondo and K. Tanimoto. "Synergistic use of adjoint-based topology and shape optimization for the design of Bi-fluid heat exchangers". *Structural and Multidisciplinary Optimization 2022; 65:245.*
- N. Galanos, E.M. Papoutsis-Kiachagias, K.C. Giannakoglou. "The Cut-Cell Method for the Conjugate Heat Transfer Topology Optimization of Turbulent Flows Using the ``*Think Discrete-Do Continuous*" Adjoint". Energies 2024; 17(8):1817.
- N. Galanos, E.M. Papoutsis-Kiachagias, K.C. Giannakoglou. "A continuous adjoint cut-cell formulation for topology optimization of bi-fluid heat exchangers". International Journal of Numerical Methods in Heat & Fluid Flow 2025 (to appear).
- K.C. Giannakoglou, E.M. Papoutsis-Kiachagias, Th. Skamagkis, A .-S.I. Margetis, N. Galanos and M. Monfaredi. "The Continuous Adjoint Method in Aero/Hydrodynamic Optimization, incl. Conjugate Heat Transfer." *VKI Lectures Series on Opt. Methods for CFD*, May 16-20, 2022.
- K.C. Giannakoglou, V.G. Asouti, E.M. Papoutsis-Kiachagias, N. Galanos, M.G. Kontou and X.S. Trompoukis. "The Think Discrete-Do Continuous adjoint in aerodynamic shape optimization". EUROGEN 2023, 15th International Conference on Evolutionary and Deterministic Methods for Design, Optimization and Control, Chania, Greece, June 1-3, 2023.
- N. Galanos, E.M. Papoutsis-Kiachagias and K.C. Giannakoglou. "A Continuous Adjoint Cut-Cell Formulation for Topology Optimization of Fluid Systems with one or two Fluids and Conjugate Heat Transfer." Advances in Computational Heat and Mass Transfer. ICCHMT 2023. Lecture Notes in Mechanical Engineering, Springer, 2024.