Interdepartmental Postgraduate Studies Program “Computational Mechanics”

Computational Techniques & Solution Algorithms

FORMULA SHEET (for the exams)
(In the exams you will not have access to books or notes.
You may only bring this formula sheet. with you)

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Approximate Factorization methods SIP & MSIP

Stencil for node (i,j):

C	F	K
B	E	H
A	D	G

Corresponding to

I-1,J+1	I,J+1	I+1,J+1
I-1,J	I,J	I+1,J
I-1,J-1	I,J-1	I+1,J-1

The SIP algorithm for 9-diagonal matrices (capital letters stand for its diagonals) computes the Lower (diagonals: a,b,c,d,e) and Upper (unit entries in the main diagonal; other diagonals: f,g,h,k) triangular matrices, according to the formulas:

$$a_{i,j} = A_{i,j}$$

$$b_{i,j} = \frac{B_{i,j} - \psi f_{i-,j+1} C_{i,j} - a_{i,j} f_{i-1,j-1}}{1 - \psi f_{i-1,j} f_{i-1,j+1}}$$

$$c_{i,j} = C_{i,j} - b_{i,j} f_{i-1,j}$$

$$d_{i,j} = \frac{D_{i,j} - 2\psi a_{i,j} g_{i-1,j-1} - a_{i,j} h_{i-1,j-1} - b_{i,j} g_{i-1,j}}{1 + 2\psi g_{i,j-1}}$$

$$e_{i,j} = E_{i,j} - a_{i,j} k_{i-1,j-1} - b_{i,j} h_{i-1,j} - c_{i,j} g_{i-1,j+1} - d_{i,j} f_{i,j-1} +$$
$$+ 2\psi (c_{i,j} f_{i-1,j+1} + d_{i,j} g_{i,j-1}) + \psi (a_{i,j} g_{i-1,j-1} + c_{i,j} k_{i-1,j+1})$$

$$f_{i,j} = \left(F_{i,j} - 2\psi c_{i,j} [f_{i-1,j+1} + k_{i-1,j+1}] - b_{i,j} k_{i-1,j} - c_{i,j} h_{i-1,j+1}\right) / e_{i,j}$$

$$g_{i,j} = \left(G_{i,j} - d_{i,j} h_{i,j-1}\right) / e_{i,j}$$

$$h_{i,j} = \left(H_{i,j} - \psi d_{i,j} g_{i,j-1} - d_{i,j} k_{i,j-1}\right) / e_{i,j}$$

$$k_{i,j} = K_{i,j} / e_{i,j}$$

The restarted GMRES method:

Solution of the linear system $Ax = q$ ($r^n = b - Ax^n$); selected basis size=m. In each generation (n+1), the solution is updated as

$$x^{n+1} = x^n + \sum_{i=1}^{m} \beta_\imath v^i = x^n + U^m B_m$$

where B_m is the column arrays of $\beta_\imath$. The following two equations are valid

$$AU^m = U^m H_m + w^m e_m^T = U^{m+1} \overline{H}_m$$

and

$$\left(U^m\right)^T AU^m = H_m$$

(notations as in the course)/

The computation of array B results from the minimization of $\left\| r^0 - U^{m+1} \overline{H}_m B_m \right\|_2$.

The Arnoldi algorithm, in two variants, follows:

Variant (A1)	Variant (A2)
$v^1 = \dfrac{r^0}{\left\|r^0\right\|}$	$v^1 = \dfrac{r^0}{\left\|r^0\right\|}$
DO j=1,m	**DO j=1,m**
$w^j = Av^j$	$w^j = Av^j$
do i=1,j	**do i=1,j**
$h_{ij} = (w^j, v^i)$	$h_{ij} = (w^j, v^i)$
enddo	$w^j = w^j - h_{ij} v^i$
$w^j = w^j - \sum_{i=1}^{j} h_{ij} v^i$	**enddo**
$h_{j+1,j} = \left\|w^j\right\|_2$	$h_{j+1,j} = \left\|w^j\right\|_2$
$v^{j+1} = \dfrac{w^j}{h_{j+1,j}}$	$v^{j+1} = \dfrac{w^j}{h_{j+1,j}}$
ENDDO	**ENDDO**